PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Quadrant Electrometers

W. E. Ayrton, J. Perry and W. E. Sumpner

Phil. Trans. R. Soc. Lond. A 1891 **182**, 519-564
doi: 10.1098/rsta.1891.0013

XIII. *Quadrant Electrometers.*

By W. E. AYRTON, J. PERRY, *and* W. E. SUMPNER.

Received May 19,—Read June 4, 1891.

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I.—GENERAL ACCOUNT OF THE EXPERIMENTS.

IN 1886, on continuously charging up the needle of Sir WILLIAM THOMSON'S bifilar quadrant electrometer, No. 5, made by Messrs. WHITE, of Glasgow, and in use at the laboratories at the Central Institution, it was noticed that the deflection of the needle, when the same P.D. (potential difference) was maintained between the quadrants, instead of steadily increasing, first increased and then diminished, so that both for a large charge on the needle as well as for a small, the sensibility of the instrument was small. A similar effect had been described by Dr. J. HOPKINSON in the 'Proceedings of the Physical Society,' vol. 7, Part I., for the previous year, and the explanation he gives of this curious result is that if the aluminium needle be below the centre of the quadrants the downward attraction of the needle, which increases with the square of the needle's charge, increases the pull on the bifilar suspension, and so for high charges more than compensates for the increased deflecting couple due to electrical action. On raising, however, the needle of our electrometer much above the centre of the quadrants, the anomalous variation of sensibility of the instrument, with increase of charge in the needle, did not disappear, and even when the needle was raised so that it was very close to the top of the quadrants, and when, if Dr. HOPKINSON's explanation were correct, the sensibility (or deflection corresponding with a given P.D. between the quadrants) ought to have been very great for a large charge on the needle, it was, on the contrary, found to be small.

As the needle had been somewhat damaged while the instrument was in the possession of the late Mr. CROMWELL VARLEY, before it was so kindly made over to the Central Institution by his son, Mr. CROMWELL VARLEY, Junior, together with other apparatus belonging to his father, a new needle, platinum wire, and weight, were obtained from Messrs. WHITE. On suspending the new needle with the fine platinum wire and weight, as received from Messrs. WHITE, the needle was found to have two modes of vibration, one the ordinary slow one, when the platinum weight turned in the acid at the bottom of the Leyden jar, and the other a very quick one, due to the twisting of the wire itself without the weight moving.* This quick vibration was removed by replacing the fine platinum wire supplied by Messrs. WHITE with a somewhat thicker one. The quadrants were adjusted for symmetry, the silk fibres tightened so as to have equal tension, as shown by the sensibility for a given potential of the needle, being a minimum, and the electrometer again tried, but it was still found that when the quadrants were close to one another, and when, therefore, as the needle was best shielded from external action, the standard formula for the electrometer might be expected to be most nearly fulfilled, the sensibility as before, first increased as the charge in the needle was increased, and then steadily diminished for further increase of charge on the needle. It was now, however, observed that, if the distance separating the quadrants was increased to $\frac{1}{4}$ inch, the sensibility tended to a limit for a large charge instead of first increasing and then diminishing.

Before replacing the new needle, it was carefully weighed with the platinum wire attached and the weight dipping into the acid, and a calculation was made as to the magnitude of the effect that should arise from the change of the pull of the fibres due to any upward or downward attraction of the needle by the quadrants. This calculation showed that for a P.D. of 3000 volts between the needle and the quadrants, the amount of such attraction was quite unable to account for the observed diminution of sensibility with large charges in the needle. Dr. HOPKINSON says in his paper, "Increased tension of the fibres from electrical attraction does not therefore account for the whole of the facts, although it does play the principal part." The experiments that we made at the end of 1886 and beginning of 1887, confirmed by the calculation above referred to, proved that, at any rate in our specimen of the quadrant electrometer, the principal part of the anomalous action was not caused by an increased tension of the fibres, and that, therefore, some other cause must be looked for to explain the observed results. The following investigation was, therefore, undertaken to ascertain the cause, and the means of removing it, and we are happy to say that it

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* We do not know whether the weight, with the fine wire attached, supplied us by Messrs. WHITE is exactly the same as is supplied generally by them with the quadrant electrometer, if so the users must experience a very unnecessary difficulty from the Blackburn-pendulum-like vibration of the needle.

has resulted in our finding out that a quadrant electrometer, as made by Messrs. WHITE can, by a special adjustment being given to its quadrants, be made to fulfil the law given in text books for the quadrant electrometer; secondly, that this special adjustment may be dispensed with if the construction of the instrument be altered; and thirdly, that when this alteration in the construction is carried out, it is possible to obtain an instrument having a sensibility many times as great as that obtainable with the White form of construction.

As long as an electrometer is only employed to measure a steady P.D. between the quadrants, it may not be of much interest to know the way in which the sensibility varies with the charge in the needle, unless great sensibility be required; in which case it is important to know for what charge on the needle maximum sensibility is obtained; in fact, for the measurement of steady P.D's. the main thing to be sure of, is that the deflection is proportional to the P.D. between the quadrants when the charge in the needle is constant. But when the electrometer is employed in conjunction with a non-inductive resistance to measure the power given by an alternate or an intermittent current, which may be any function of the time, to a circuit that may possess capacity, or an E.M.F., or self or mutual induction, or all four (in accordance with the method devised by Professor FITZGERALD and ourselves during the meeting of the Electrical Congress in Paris in 1881, and published the same autumn by M. POTIER), it is absolutely necessary that the electrometer should exactly fulfil the law—

$$t = k\,(Q_1 - Q_2)\left(N - \frac{Q_1 + Q_2}{2}\right),$$

where t is the torque exerted at any moment on the needle by electrical action, and Q_1, Q_2, and N the potentials, at the same time, of the opposite pairs of quadrants and of the needle respectively relatively to the outside case of the instrument, and k is a constant. [See Section VI. "Electrometer used to Measure Power."]

Much has been written during the last three or four years by MM. GOUY, LEDEBOER, and others regarding the departure of the action of the quadrant electrometer from the law given above, but we are not aware that hitherto the cause of the discrepancy has been pointed out, or the means of removing it and obtaining an instrument that fulfils this law.

We thought at first that the curious results might be due to some capillary action between the platinum weight and the sulphuric acid which varied with the potential of the weight, but, experiments having shown that no such action existed, it was thought desirable to measure the P.D. between the needle and the outside case of the instrument by means of one of Sir WILLIAM THOMSON'S absolute electrometers (made especially sensitive by thinning the little coach springs carrying the movable aluminium disc), and so ascertain completely the laws connecting the variation of sensibility of the quadrant electrometer with the potential of the needle, the distance between the silk fibres, the distance apart of the quadrants, &c. The investigation has taken some years to complete, and in carrying it out this quadrant electrometer has had to be taken entirely to pieces many times. The labour that this has entailed will be appreciated by anyone who has had experience in taking a White electrometer to pieces.

In the ordinary form of the quadrant electrometer as constructed by Messrs. WHITE it is not possible to get at the guard-tube or the lower part, either of the needle, or of the quadrants, without taking the whole interior mechanism bodily out of the Leyden jar. Since in doing this there is danger of breaking the silk fibres from which hangs the aluminium needle with its attached platinum weight, especially as the pull on the fibres is, of course, much increased when the weight is lifted out of the dense sulphuric acid, it may be advisable to clamp the needle and take the stress off the fibres before lifting out the interior of the instrument. And, if, for example, it be desired to adjust the aluminium wire relatively to the guard-tube in which it hangs, or to remove this guard-tube so as to be able to get at the attachment of the aluminium wire, or to straighten the needle if slightly bent, or, generally, to make any adjustment liable to break the silk fibres, it is certainly advisable to clamp the needle before removing the interior of the instrument. Under these circumstances the following operation has to be performed :—1st. Clamp the needle, lift the interior of the apparatus out of the Leyden jar, and place it on some metallic support provided with levelling screws by which the instrument can be levelled. 2nd. Make the necessary adjustment, and unclamp the needle to see whether the adjustment has been correctly made. 3rd. Replace the interior of the apparatus in position inside the Leyden jar.

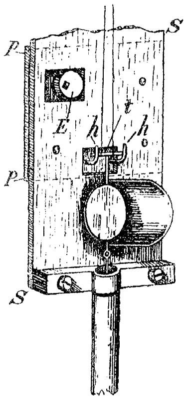

Fig. 1.

The method ordinarily adopted for clamping the needle consists in passing a screw through a loop in the aluminium wire connecting the mirror with the needle, and screwing this screw into the insulated framework supporting the apparatus.

There is not much difficulty in putting in this screw by means of the square headed screwdriver provided with the instrument, but there is the greatest difficulty in removing the screw without breaking the fibres. In fact, the danger of breaking the fibres when making one of the adjustments above referred to without previously clamping the needle is hardly greater than the danger of breaking them when removing the clamping screw.

We, therefore, had fitted to the electrometer the device which is seen in fig. 1* for easily raising and lowering the needle. *E* is a circular piece of metal turning eccentrically about a pin fixed to a small plate *pp*, which can slide up and down at the back of the insulated support *SS*. This plate *pp* carries two little hooks *hh*, which, on being raised by turning the eccentric *E* (by means of the square screwdriver provided with the electrometer), lift up the T-piece *t* at the top of the suspension, and take the weight of the mirror, the needle, and its attachment, off the silk fibres. When then it is desired to remove the interior of the electrometer out of the Leyden jar, all that is necessary to be done is to turn the eccentric *E* one way to raise the needle, and the other way to lower it, either of which operations can be done quickly without the slightest risk of breaking the fibres.

If the instrument is to be packed for travelling it will be well to also clamp the needle by means of the screw inserted as usual in the loop on the aluminium wire, but as the eccentric lifting arrangement which we have added to the electrometer enables the screw to be inserted and removed, if it be desired to use it, when there is no tension whatever on the fibres, the clamping and unclamping by means of the screw can be done with perfect safety. We have to thank Mr. H. B. Bourne for helping us in designing this eccentric attachment for the White electrometer.

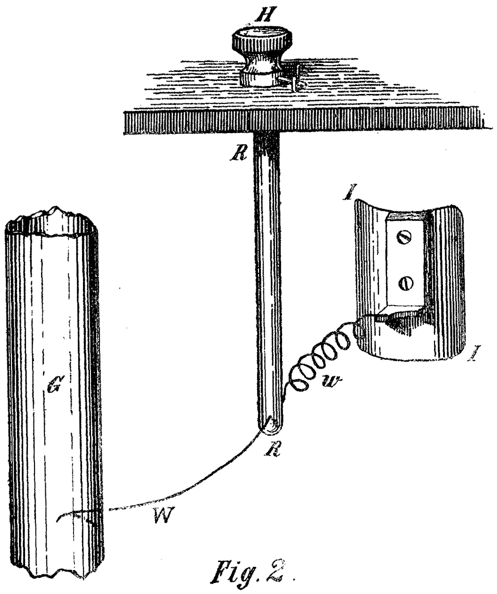

Fig. 2.

We may take this opportunity of mentioning another modification that we have

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* In order that the mirror may be more clearly seen, one half of the horizontal guard-tube which surrounds the mirror has been omitted in fig. 1.

introduced, with the assistance of Mr. BOURNE, for the purpose of improving the good working of the quadrant electrometer. In the instrument as ordinarily constructed the framework of the replenisher is always in connection with the sulphuric acid which forms the interior coating of the Leyden jar, and hence electric leakage can always take place over the framework of the replenisher. To avoid this we altered the replenisher in one of our quadrant electrometers, so that by raising a knob on the top outside the instrument the replenisher could be bodily raised up until the platinum wire attached to it was lifted out of the acid. But we think that the following very simple device which we have attached to another electrometer is in every way better. Instead of the replenisher being electrically connected with the interior of the Leyden jar through the acid, as is the case with Messrs. WHITE's instruments, we arrange that it can be connected with, or disconnected from, the guard-tube G (fig. 2), which is itself always in connection with the acid. This is done thus :—II, the insulated inductor of the replenisher, is attached by means of a flexible wire to a springy wire W carried at the end of an ebonite rod RR. By turning the milled head H this springy wire W can be made to touch, or not to touch, the guard-tube, and thus II can be connected with the interior of the Leyden jar, when the replenisher is to be used, or left wholly disconnected from the Leyden jar, so that there is absolutely no leakage over the replenisher frame, when the replenisher is not in use.

With this device we can dispense altogether with the clamping arrangement that is usually employed for holding the rotatory spindle that supports the carriers of the replenisher in such a position that the carriers are not in contact with the inductors.

In the ordinary use of the quadrant electrometer the charging rod is kept disconnected from the Leyden jar, and therefore the insulation of the charging rod need not be very perfect, since leakage can only occur over it during the time the Leyden jar is being charged ; in these experiments, however, as the interior of the Leyden jar of the quadrant electrometer and the needle had to be permanently connected with the lower movable disc of the absolute electrometer, it was necessary to keep the charging rod of the quadrant electrometer always in connection with the Leyden jar of this instrument. This made the experiments very difficult to perform on damp days, indeed on some days hours have been spent without a single trustworthy result being obtained. Hence the results given in the following tables, which are the trustworthy ones, only represent a small portion of the work that has been actually carried out.

As the two electrometers are permanently fixed on two brick pillars, some 14 feet apart, in the laboratory, the absolute electrometer being close to the light, the reflecting quadrant electrometer with its scale well away from the light, the connection between the lower disc of the absolute and the charging rod of the quadrant electrometers was made by a long fine wire supported on a well insulated glass rod, shielded from dust and kept dry by sulphuric acid. The outsides of the two instruments were also connected together by an uninsulated copper wire.

In the earlier of the experiments the Leyden jars of the two electrometers were charged quite irrespectively of one another, sometimes positively, and sometimes negatively, but as the investigation advanced both jars were always charged positively, the P.D. between the inside and outside of the jar of the absolute electrometer being maintained at + 5800 volts, and the force required to bring the movable aluminium disc to the sighted position was always the same, as the reading of the upper micrometer screw was not varied. The attracted aluminium disc was brought to the sighted position by moving the lower plate only, and the potential of the needle of the quadrant electrometer was measured by showing the difference between the readings of the screw moving this lower plate (1) when the lower plate was connected with the outside of the absolute electrometer; (2) when the lower plate was disconnected from the case and connected with the needle of the quadrant electrometer. The absolute electrometer behaved well when the upper micrometer screw was not touched, the earth reading for a fixed P.D. between the inside and outside of the Leyden jar varying but very slightly from day to day.

A series of calibrations of this absolute electrometer carried out in the ordinary way by placing known weights on the movable aluminium disc to reproduce the effect of electric attraction, showed that with the position we gave to the upper micrometer screw, and taking "v" as 30 true ohms, a motion of the lower plate through one division of the vertical scale corresponded with 358·6 volts.

In July, 1888, several large P.D.'s were measured by means of this absolute electrometer (using the constants that we had determined for this instrument), and also by means of one of Sir WILLIAM THOMSON's commercial "electrostatic voltmeters" reading to 20,000 volts, kindly lent us by Messrs. ELLIOTT BROS. The result of these comparisons led first to a correction in the constants that had been previously sent out with the electrostatic voltmeters from Glasgow, and secondly to a new determination of the value of "v." For Sir WILLIAM THOMSON had calibrated these voltmeters by a galvanometer method, based on the value of the electro-chemical equivalent of silver, as determined by Lord RAYLEIGH and Mrs. SIDGWICK,* while we had checked the calibration of the electrostatic voltmeter, by comparing this instrument with the absolute electrometer. The value of "v" thus obtained was 298,000,000 metres per second.

The scale employed with the quadrant electrometer is a circular one at 5 feet distance from the mirror. There are 290 divisions in 1 foot length of scale, and as both scale and electrometer are permanently screwed down in position, one division of the scale always corresponds with $\frac{1}{2900}$ radian deflection of the mirror, or about $\frac{1}{50}$ of a degree. The scale is so placed that the mechanical zero of the electrometer corresponds with 0 on the scale, that is to say, the spot of light is at 0 when the needle is adjusted as symmetrically relatively to the quadrants as it is possible to do with the

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* "Electrometric determination of v," 'Brit. Assoc. Report,' 1888. See also the 'Electrician,' p. 448, vol. 22, 1889, for the correction for the effective area of the attracted disc in the absolute electrometer.

eye, the whole instrument being discharged. The *electrical* zero is next made to coincide with the *mechanical* zero, that is to say, on charging the needle when the four quadrants are connected with the outside case of the instrument, the fourth quadrant is slightly moved until the spot of light is brought back to the same place that it occupied on the scale when the needle was discharged. But it was found that if this were done for some particular potential of the needle, a fresh adjustment of the fourth quadrant had to be made for each potential to which the needle was charged, or, in other words, it was practically impossible so to adjust the quadrants that without further touching them the electrical and mechanical zeros should be the same for a considerable variation of the potential of the needle. Hence, in what follows the position of the electrical zero is in each case given. The question of the electrical and mechanical zeros differing from one another will be further dealt with in Section V., entitled "Motion of the Electrical Zero."

II.—EFFECT OF VARYING THE DISTANCE BETWEEN THE FIBRES.

If N be the P.D. in volts between the needle and the outside case of the electrometer, and D the deflection of the needle in microradians produced by a P.D. equal to V between the opposite pairs of quadrants, one pair being connected with the outside case, the ordinary formula given for the electrometer reduces to

$$D = \frac{\sigma}{2} V \left(N - \frac{V}{2} \right),$$

where σ is usually regarded as being constant both for variations of V and of N, the distance between the quadrants and the distance between the fibres being kept constant.

The first thing tested was the constancy of σ for a fixed value of N and for varying values of V. The relative values of V were measured by means of a Thomson's and Varley's potential divider, which enables a P.D. to be accurately subdivided to the $\frac{1}{10000}$th part if required, and it was found that for any large and fixed charge on the needle less than the value which produced instability, and for any fixed position of the quadrants and distance between the fibres, the deflection of the needle D was proportional to V to within 2 per cent., that is to say, when V and D alone were the variables σ was always constant to within 2 per cent. In the great majority of cases the constancy of σ was much closer than this, but, as it was found necessary to vary the position of the fourth quadrant in order that the deflections for all the different values used for V should be on the scale when the charge in the needle was varied, the want of constancy of σ for unsymmetrical positions of the instrument was sometimes nearly 2 per cent.

On Sheet I. (Plate 9) are seen two curves connecting the double deflection of the electrometer needle with the deflecting P.D. between the quadrants. One set of results was obtained when the P.D. between the needle and the outside case of the instrument was maintained at 1793 volts, and the other when the needle's potential was 2510 volts. In each case the curve is practically a straight line, and it will be observed that in spite of the needle's potential having been increased about 40 per cent., the combination of these two lines is practically the same.

The next point tested was the constancy of σ when N was varied. When N is large the preceding formula is approximately

$$D = \frac{\sigma}{2} VN ,$$

so that

$$\sigma = \frac{2D}{VN} .$$

In the following tables σ is the deflection of the ray of light in *microradians*, divided by the product of the deflecting P.D. in volts maintained between the quadrants into the P.D. in volts between the needle and the case of the electrometer. σ is thus the *sensibility factor* of the instrument for a given position of the quadrants, and for a given distance between the fibres; and the different values of σ not only show how the behaviour of this electrometer departs from the simple mathematical law, but enables its sensibility to be compared with the sensibility of any other quadrant electrometer which may be tested *absolutely* in this manner.

On November 3, 1887, the fibres were adjusted so as to be parallel, and at about 2 millims. apart, and the results of the experiments are given in Table I. The values for the double deflection per one Leclanché cell for the different values of N are also plotted in the curve on Sheet II. (Plate 9), corresponding with "Fibres Parallel." It is interesting to notice that while σ is far from being constant for different values of N it has the same value for the same value of N, although V and D are very different, thus confirming the experiments previously made with fixed values of N and different values of V.

TABLE I.—November 3, 1887. Fibres Parallel.

Deflecting P.D. in volts.	Potential of needle in volts.	Electrical zero.	Double deflection per Leclanché cell.	Sensibility factor.	Remarks.
1·45	1886	30	130	16·4	Adjustable quadrant at +100.
	1543	20	118	17·9	
	1219	10	104	20·3	
	944	..	86	21·5	
	705	..	66	22·4	
	466	..	43	22·0	
	158	0	14	21	The potential of the needle could not be very accurately measured when less than about 360 volts.
	1513	− 20	118	18·6	
	2733	− 120	154	13·4	
	2378	− 50	138	13·9	Points marked ⊙ on the curve, Sheet II. (Plate 9).
	1914	− 30	128	15·4	
2·9	2726	− 150	152	13·3	Adjustable quadrant at + 92·5.
	2259	− 60	140	14·7	
	1919	− 40	128	16·0	
	1822	− 35	126	16·4	
	1531	− 25	115	18·0	
	1305	− 20	104	18·9	
	1201	− 18	99	19·6	
	1015	− 15	87	20·4	Points marked ◬ on the curve, Sheet II. (Plate 9).
	914	− 12	84	21·8	
	792	− 11	70	21	
	638	− 9	59	22	
	599	− 9	54	21·4	
	398	− 5	36	21·6	
	262	− 3	25	..	The potential of the needle could not be very accurately measured when less than about 360 volts.
	204	− 4	18	21·0	
	129	− 2	7	..	
1·45	1951	29	127	15·1	Adjustable quadrant at 113.
	746	13·5	70	22·4	
	1161	22	99	20·2	
	1560	29·5	120	18·3	
	2636	8	147	13·3	
	2621	10	148	13·4	Points marked × on curve, Sheet II. (Plate 9).
	2392	24	143	14·3	
	2435	20	145	14·2	
	2199	28	119	..	
	1829	31	127	16·5	
	610	9	57	22·3	

The fibres were now put nearer together at the top, and the experiments repeated. The results obtained are given in Table II. and are plotted on the curve on Sheet II. (Plate 9), corresponding with "Fibres near at top." The departure from the law that the sensibility of the electrometer is proportional to the potential of the needle, *i.e.*, the want of constancy of the "sensibility factor" is here very marked, a maximum sensibility being obtained when the P.D. between the needle and the metallic case of the electrometer is about 1800 volts.

TABLE II.—November 4, 1887. Fibres near together at the Top.

Deflecting P.D. in volts.	Potential of needle in volts.	Electrical zero.	Double deflection per Leclanché cell.	Sensibility factor.	Remarks.
1·45	2420	0	176	17·3	Adjustable quadrant at 160.
	2223	20	187	20	
	2080	30	192	21·9	
	1919	38	195	24·2	
	1768	40	197	26·5	
	1606	43	198	29·3	
	1506	45	193	30	
	1338	44	191	33·9	
	1112	37	180	38·5	Points marked ⊙ on the curve, Sheet II. (Plate 9).
	961	35	170	42	
	889	31	162	43·3	
	753	21	146	46·1	
	674	18	137	48·3	
	581	15	120	49·1	
	484	9	103	50·6	
	376	5	81	51·2	
	255	2	58	54·2	
December 3, 1887.					
	240	1	49	48·5	The potential of the needle could not be very accurately measured when less than about 360 volts.
	359	2·5	70	46·4	
	538	7	100	44·2	
	717	13	133	44·1	
	896	20	156	41·4	
	1076	26	174	38·5	
	1255	31	185	35·1	Adjustable quadrant at 50.
1·45	1434	36	194	32·2	Points marked × on the curve, Sheet II. (Plate 9).
	1610	39	199	29·4	
	1793	42	201	26·7	
	1972	42	202	24·4	
	2152	41	200	22·1	
	2331	38	198	20·2	
	2510	32	195	18·5	
	2659	20	190	16·8	
	2869	8	186	16·1	
	3048	−2	184	14·4	

TABLE II. (continued).—December 4, 1887. Some Experiments repeated to Test the Accuracy of those previously made.

Deflecting P.D. in volts.	Potential of needle in volts.	Electrical zero.	Double Deflection per Leclanché cell.	Sensibility factor.	Remarks.
	717	9	136	45·1	Adjustable quadrant at +50.
	1434	32	194	32·2	
	2152	32	200	22·1	Points marked × on the curve, Sheet II. (Plate 9).
	2869	−5	187	15·5	

From the dates given for the tests contained in Table II., it will be seen that this group was made at intervals lasting over one month. The sensibility factor is very far from being constant, varying from about 14 when the potential of the needle was about 3000 volts to about 50 when it was about 300 volts; but nevertheless for each potential of the needle with the constant distance between the quadrants, and between the fibres, the sensibility has practically a perfectly constant value. The following examples selected from Table II. illustrate this:—

Date.	Potential of needle in volts.	Sensibility factor.
Nov. 4	753	46·1
Dec. 3	717	44·1
Dec. 4	717	45·1
Nov. 4	1506 1338 } Mean 1444	30 33·9 } Mean 32
Dec. 3	1434	32·2
Dec. 4	1434	32·2
Nov. 4	2080	21·9
Dec. 3	2152	22·1
Dec. 4	2152	22·1
Nov. 4	2420	17·3
Dec. 3	2869	16·1
Dec. 4	2869	15·5

The small differences between the values of the sensibility factor on November 4, and on the later dates for the same potential of the needle, arises probably from the fact that in the interval the fibres were separated far apart at the top to enable the next set of experiments to be made, and approached again on December 3, until they were again, as far as the eye could judge, in the same position as on November 4. In changing the distance between the fibres, as well as in altering the tension of the fibres to adjust in each case for minimum sensibility (corresponding with equal tension) only one of the rollers on which the fibres are wound was touched, in order that a position could be reproduced as nearly as possible. But it was, of course, impossible to exactly reproduce a previous position of the fibres, and the sensibility factor for a fixed potential of the needle, and for a so-called fixed position of the fibres was, consequently, not absolutely constant.

In view, however, of the extreme delicacy of the tests and the great variation of the sensibility factor with a variation of the potential of the needle, the wonder is that for the same potential of the needle the sensibility factor was as constant as it was.

On November 8, 1887, the fibres were separated until they were far apart at the top, the tension adjusted for minimum sensibility, and experiments were made on November, 8, 10, 11, and December 2, to determine the values of the sensibility factor. The results are given in Table III., and the values of the double deflection per Leclanché cell for the different P.D's. between the needle and the outside case are plotted in the curve on Sheet II. (Plate 9), corresponding with "Fibres far apart at the Top."

TABLE III.—November 8, 1887. Fibres far apart at the top.

Deflecting P.D. in volts.	Potential of needle in volts.	Electrical zero.	Double deflection per Leclanché cell.	Sensibility factor.	Remarks.
4·35	2256	— 13	89·6	9·4	Adjustable quadrant at 68.
	1936	— 6	81	9·9	
	1596	— 2	71	10·6	
	1338	— 1	63·6	11·3	Points marked ⊙ on the curve, Sheet II. (Plate 9).
	1047	— 0·5	52·3	11·9	
	778	0	41·3	12·6	
	513	0	29	13·4	
	323	0	19	14	
November 10, 1887.					
2·9	359	— 1	19	12·6	Adjustable quadrant at — 50. Points marked ◬ on the curve, Sheet II. (Plate 9).
	717	— 4	38	12·6	
	1076	— 8	53	11·7	
	1434	— 15	67	11·1	
	359	2	18·5	12·3	Adjustable quadrant at 0.
	717	8	39	12·9	
	1076	16	53·5	11·8	
	1434	28	68·5	11·3	
November 11, 1887.					
2·9	2869	— 42	110	9·1	Adjustable quadrant at — 100.
	2510	— 50	93	8·8	
	1793	— 35	75	9·9	
	1076	— 18	54	12	Points marked ◬ on the curve, Sheet II. (Plate 9).
	2152	— 43	84	9·3	
	2869	— 42	106	8·8	
	3048	— 35	118	9·2	
December 2, 1887.					
1·45	2252	— 29	87	9·2	Adjustable quadrant at — 100. Weather unfavourable, leakage rather rapid. When the potential of the needle was high the needle was drawn aside, causing the electrical zero to differ much from the mechanical zero, and rendering the sensibility factor unusually small. When this happened, a further increase in the potential of the needle caused it to stick. The experiment with the needle at a potential of 3184 volts is an example of this. Points marked × on the curve, Sheet II. (Plate 9). *Needle stuck.
	2887	— 27	111	9·1	
	2986	— 20	118	9·4	
	3077	— 15	133	10·3	
	3023	— 13	124	9·9	
	3184	— 100	64	4·8	
	3137	— 33	108	8·2	
	3087	+ 2	136	7·3	
	3055	— 8	127	9·9	
	2900	— 26	108	8·9	
	2840	— 29	105	8·8	
	3045	— 10	124	9·7	
	3077	— 8	127	9·8	
	1832	— 18	76	9·9	
	2180	— 26	84	9·2	
	2714	— 33	97	8·5	
	2883	— 30	104	8·6	
	3163	+ 8	138	10·4	
	3586*	..	..	..	

From the Tables of Results and from the curves on Sheet II. (Plate 9), we see that when the fibres are parallel the sensibility factor falls off somewhat as the potential of the needle is increased, whereas with the fibres far apart at the top the sensibility factor first falls off, and then increases again as the potential of the needle steadily increases. With the fibres near together at the top the curve is perhaps the most interesting, as here the sensibility factor falls off so much for high potentials of the needle that the deflection of a fixed P.D. betweeen the quadrants is actually smaller for a high potential of the needle than for a lower potential.

III.—EFFECT OF VARYING THE DISTANCE BETWEEN THE QUADRANTS.

The reasoning which leads to the ordinary formula for the quadrant electrometer

$$D = \frac{\sigma}{2} V \left(N - \frac{V}{2}\right)$$

(where D is the deflection produced by a P.D. equal to V between the opposite pairs of quadrants, one pair being connected with the outside case of the instrument, and N the P.D. between the needle and the outside case), shows that σ is proportional to the variation of capacity of the needle per radian deflection. Consequently it might be expected that since the needle is less covered by the quadrants when they are drawn out, and, since the variation of capacity of the needle per radian deflection is less when the quadrants are pulled out than when they are in, that the instrument would be less sensitive when the quadrants are drawn out than when they are in. But those who are accustomed to use the quadrant electrometer know that the opposite is the case. Here again then there is a marked difference between the actual behaviour of the instrument, and the behaviour that the simple formula would lead one to expect.

In all the preceding experiments three of the quadrants had been left untouched with their edges at from $2\frac{1}{2}$ to 3 millims. apart, and the fourth quadrant, the one to which the micrometer screw is attached, had alone been moved. But at the beginning of 1888 experiments were made to determine the way in which the sensibility factor varied with the potential of the needle for different distances between the quadrants and for different distances between the fibres. When the quadrant under the induction plate was moved far out, it seemed quite possible that the latter would exert some influence on the curves. The deflections, however, were found to be practically unaffected by disconnecting the induction plate from one pair of quadrants, and connecting it with the other pair. As a rule the induction plate was kept electrically connected with the quadrant immediately underneath it. The distances between the quadrants given in the following tables were estimated by eye only, and are, therefore, only approximately correct.

Table IV. gives the results for four positions of the quadrants when the fibres were near together, and the curves on Sheet III. (Plate 10) show the results recorded graphically.

The quadrants are called respectively Q_1, Q_2, Q_3, and Q_4; Q_4 being the one adjustable by means of the adjustable screw.

TABLE IV.—January 23, 1888. Fibres near together at the top.

Quadrants about $2\frac{1}{2}$ millims. apart.					
Deflecting P.D. in volts.	Potential of needle in volts.	Electrical zero.	Double deflection per Leclanché cell.	Sensibility factor.	Remarks.
1·45	359	1	80	53·2	Points marked × on the curve, Sheet III. (Plate 10).
	717	3	134	44·6	
	1075	7	167	37·5	
	1434	8	179	29·8	
	1793	+ 7	182	24·2	
	2152	− 1	175	19·3	
	2510	− 17	166	17·6	
	2869	− 40	162	13·5	
	3227	− 86	157	11·6	
	3407	− 120	160	11·2	
	3586	− 165	175	11·6	
Quadrants about $3\frac{1}{2}$ millims. apart.					
1·45	1076	. .	182	40·3	Points marked ⊙ on the curve, Sheet III. (Plate 10).
	1793	33	220	29·3	
	2510	− 8	209	19·9	

January 24, 1888.

Deflecting P.D. in volts.	Potential of needle in volts.	Electrical zero.	Double deflection per Leclanché cell.	Sensibility factor.	Remarks.
Quadrants about $3\frac{1}{2}$ millims. apart.					
1·45	359	6	80	53·2	Points marked ⊙ on the curve, Sheet III. (Plate 10) ⊙.
	717	12	142	47·0	
	1076	21	181	40·2	
	1434	26	207	34·3	
	1793	25	218	29·0	
	2152	16	217	24·0	
	2510	− 3	212	20·2	
	2869	− 40	205	17·0	
	3227	− 102	215	15·9	
	3586	− 277	255	17·0	
Quadrants about 4 millims. apart.					
	1793	− 11	..		
	1434	2	332	54·2	Points marked ◬ on the curve, Sheet III. (Plate 10).
	1255	3	279	52·9	
	1076	3	235	52·1	
	896	3	196	52·2	
	538	1	121	53·5	
	1793	5	331	44	Quadrant Q_1 pushed inwards very slightly, the three other quadrants remaining untouched. Points marked ▽ on the curve, Sheet III. (Plate 10).
	1076	19	216	47·8	
	2152	− 23	370	41·0	
	2331	− 43	369	37·7	
	2331	..	376	38·3	
	2510	− 75	383	36·3	
	1793	13	290	38·5	Q_3 pushed inwards very slightly, and Q_4 slightly altered to bring electrical zero to near the middle of the scale, the other quadrants remaining untouched. Points marked ⊡ on the curve, Sheet III. (Plate 10).
	2152	− 8	313	34·7	
	2689	− 82	331	29·4	
	2869	Off scale			
Quadrants about 1 millim. apart.					
	717	11	128	42·5	Points marked ⊙ on curve, Sheet III. (Plate 10).
	1076	11	145	32·2	
	1434	14	153	25·4	
	2152	11	127	14·0	
	2869	2	107	8·9	
	1434	3	142	23·6	Q_3 slightly shifted.
	717	3	128	42·5	
	2152	− 3	126	13·9	
	2869	− 27	110	9·1	
	2869	− 195	106	8·8	Q_4 slightly shifted.
	2152	− 147	126	13·9	
	1434	− 107	142	23·6	
	717	− 47	130	43·2	

An examination of the preceding results, and of the curves on Sheet III. (Plate 10), corresponding with them, shows that when the quadrants are near together the sensibility factor diminishes at first slowly and afterwards rapidly as the potential of the needle is steadily increased. As the distance between the fibres increases, the rate of diminution of the sensibility factor, with increase of the potential of the needle, grows less until, when the quadrants are about 4 millims. apart, the sensibility factor actually increases with increase of potential of the needle, so that the curve connecting deflection for a constant P.D. between the quadrants with a varying P.D. between the needle and the outside case, becomes convex instead of concave to the axis along which the potential of the needle is reckoned. Hence, with our specimen of the White electrometer, *when the distance between the quadrants is about* 3·9 *millims. the sensibility factor is a constant, and the conventional formula for the quadrant electrometer becomes the true formula.*

The next set of experiments was made with the fibres far apart at the top, the fibres being carefully adjusted for equality of tension after having been separated. The results are given on Table V., and they are also plotted on Sheet IV. (Plate 10); it will be seen that the general shape of the curves for the various distances between the quadrants is the same as was obtained when the fibres were near together.

TABLE V.—January 30, 1888. Fibres far apart at the top.

Deflecting P.D. in volts.	Potential of needle in volts.	Electrical zero.	Double deflection per Leclanché cell.	Sensibility factor.	Remarks.
Quadrants about 1 millim. apart.					
1·45	1793	− 20	88	11·7	Points marked × on the curve, Sheet IV. (Plate 10).
	2152	− 21	90	10·0	
	2510	− 23	91	8·6	
	2869	− 25	90	7·5	
	3227	− 30	92	6·7	
	3586	− 25	111	7·4	
	359	0	28	18·6	
	717	0	52	17·3	
	1076	0	71	15·7	
	1434	0	81	13·4	
	1793	− 1	88	11·7	
	2152	− 4	90	10·0	
	2510	− 5	90	8·6	
	2869	− 8	89	7·4	
	3227	− 12	91	6·7	
	3407	− 16	94	6·6	
	3586	− 11	103	6·7	
	2940	..	90	7·2	
	1434	− 1	82	13·6	
	359	0	29	19·2	
Quadrants about 2¾ millims. apart.					
1·45	1793	0	101	13·4	Points marked ⊙ on the curve, Sheet IV. (Plate 10).
	2152	− 4	110	12·2	
	2510	− 13	118	11·2	
	2869	− 38	123	10·2	
	3227	− 63	133	9·1	
	3586	− 115	147	9·8	
	359	0	27	18	
	717	1	54	18	
	1076	2	74	16·4	
	1434	2	89	14·7	
	1793	1	102	13·6	
	2510	− 12	118	11·2	
	3227	− 64	135	10	
Quadrants about 4 millims. apart.					
1·45	1793	7	144	19	Points marked ◬ on the curve, Sheet IV. (Plate 10).
	2510	− 63	237	22·5	
	3227	off the	scale		
	2152	− 5	188	20·8	
	1793	2	149	19·9	
	1076	5	85	18·8	
	359	1	30	20	

The curves contained on Sheets III. and IV. (Plate 10) show that this quadrant electrometer may be adjusted so that the variation of sensibility with the potential of the needle may be made to follow one or other of *three distinct laws.* If the quadrants be near together there are certain limits between which the potential of the needle may vary without producing more than a small change in the deflection corresponding with a fixed P.D. between the quadrants; for example, when the quadrants were about 2½ millims. apart, and the fibres near together at the top, the deflection produced by a P.D. of 1·45 volts between the quadrants only varied about 11 per cent. when the potential of the needle was increased from 896 to 3586 volts. When the fibres were far apart at the top it was when the quadrants were about 1 millim. apart as seen in Sheet III. (Plate 10) that a similar flatness was obtained in the curve connecting deflection with potential of the needle. In this case the deflection of the needle was practically quite constant when its potential varied between 2152 and 3227 volts; and even when the potential of the needle was increased from 1434 to 3407 volts, that is, by nearly 2000 volts, the deflection only increased by less than 9 per cent. This arrangement of the quadrants gives but a comparatively small sensibility, but, where great sensibility is not required, it would be a convenient one to employ, as leakage of the Leyden jar, or loss of potential of the needle due to the rapid absorption that occurs when the jar is first charged, would only slightly affect the deflection for a fixed P.D. between the quadrants.

When the quadrants were at about 3·9 millims. apart the deflection for a given P.D. between the quadrants was almost directly proportional to the potential of the needle. This then would be the arrangement to employ when the electrometer is used with alternating P.D's. And, lastly, when the quadrants were 4 or more millims. apart the deflection increased much more rapidly than the potential of the needle, so that maximum sensibility, bordering on instability, is obtained with this arrangement of the quadrants.

For each distance between the quadrants less than the distance at which instability begins to occur, the shape of the sensibility curve is perfectly definite, but its shape alters very much for an exceedingly small alteration in the position of the quadrants when the distance separating them is some three or more millims.

It might have been anticipated that the conventional mathematical law for the quadrant electrometer, viz.:—

$$D = \frac{\sigma}{2}(Q_1 - Q_2)\left\{N - \frac{Q_1 + Q_2}{2}\right\}$$

(where D is the deflection, and Q_1, Q_2, and N the potentials of the two pairs of quadrants and of the needle relatively to the outside case) would have been most nearly fulfilled when the quadrants were pushed far in, so that the needle was well under cover of them. But the preceding results show that this is very far from being the case, and that constancy in the sensibility factor σ, for different values of N, is obtained *only* when the quadrants are so far separated that the sensibility of the instrument for the particular adjustment of the distances between the fibres is fairly large.

IV.—Cause of the Peculiar Action of the White Electrometer.

Towards the end of 1887 the cause of the irregularity in the action of the Thomson quadrant electrometer, as constructed by Messrs. White, began to dawn on us. The wire supporting the aluminium needle, as well as the wire which connects the needle with the sulphuric acid in the Leyden jar, is enclosed in a metallic guard-tube to screen the wire from external action. But in order that the needle may project outside the guard-tube, GG′, openings are made in its two sides, as seen in fig. 3. Hence the moment the needle is deflected from its zero position each half of the needle becomes unsymmetrically placed relatively to the two bits I and I′ of the guard-tube. Therefore, in spite of the needle and the guard tube being always maintained at the same potential, there is a repulsion between the charges on the two bits I and I′ of the guard-tube, and the charges on the two halves of the needle; and this repulsion might not only have the defect of seriously diminishing the sensibility of the quadrant electrometer, as made by Messrs. White, but might cause the variation of sensibility of the electrometer with variation of the P.D. between the needle and the outer coating of the Leyden jar to follow a far more complicated law than that expressed by the conventional formula just given.

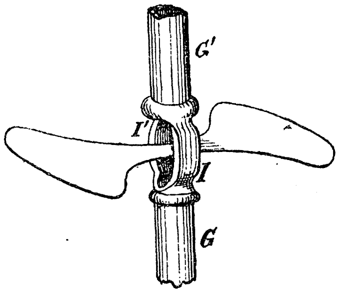


Fig. 3.

To test this theory, that the peculiarities in the law of the quadrant electrometer are due to the electric action of the guard-tube on the needle, in consequence of the special shape of the former, the want of symmetry of the guard-tube was intensified and varied by attaching a piece of thin aluminum foil to it above and below the needle, and experiments on the variation of sensibility of the electrometer with variation of the potential of the needle made for the positions of this piece of aluminium foil, F, shown in figs. 4 and 5. These figures show a section of the guard-tube through the needle, I and I′, figs. 4 and 5 giving the sections of the connecting pieces I and I′ seen in fig. 3. The difficulty connected with these experiments was very great, arising from the almost impossibility of attaching the aluminium foil to the guard-tube without slightly altering the position of the tube. And as the slightest alteration of the position of the guard-tube considerably altered the electrical zero, it was necessary in each case to readjust the movable quadrant in order that the electrical zero might coincide with the mechanical zero. This, in itself, altered the sensibility curves slightly, and the effect due to this cause must be kept in mind when comparing the sensibility curves made before attaching the foil, for the various positions of the foil, and after removing it. These experiments on the guard-tube were more troublesome than any of the others, and very great care was expended on them. Many sets of experiments were made and curves drawn to record the results; but, since in the majority of cases it was considered that too much disturbance of the position of the guard-tube had been produced in the act of attaching the foil or of altering its position, the majority of the results were rejected.

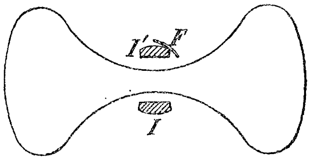


Fig. 4.

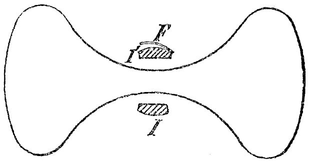


Fig. 5.

The following sets, given in Tables VI. and VII., were obtained on December 6, 7, and 8, 1887. They correspond with positions of the adjustable quadrants varying between — 50 and + 50, and, as subsequent experiments made on December 9 (the results of which are given in Table IX.) confirm the idea that this amount of variation of the adjustable quadrant alone would not account for the variations in the results that were obtained on attaching the thin piece of aluminium foil to the side of the guard-tube, and on moving this foil from the side to the back of the guard-tube, the results given in Tables VI. and VII. may be regarded as correct. They may, therefore, be compared with those contained in the latter part of Table II., and obtained on December 3 and 4 before the foil was attached, as well as with those contained in Table VIII., and obtained on December 8, after the foil had been removed. For in all these experiments the variation in the position of the adjustable quadrant was kept between the same limits — 50 and + 50, the position of the other quadrants remained the same, and there was the same distance between the fibres, which were adjusted for minimum sensibility when the potential of the needle was about 1800 volts.

The results are all plotted on Sheet V. (Plate 11), and it is important to notice that:—

1. The curves obtained with "*no foil*" were practically the same before attaching and after removing the foil, and had ordinates much higher than when the foil was attached.

2. The curves obtained with "*foil at the side of the guard-tube*" had the next higher ordinates.

3. The curves obtained with "*foil at the back of the guard-tube*" had the lowest ordinates.

We may, therefore, conclude that the attachment of a small piece of foil to the guard-tube not only altered the shape of the curve connecting the sensibility of the electrometer with the potential of the needle, but, whether the foil was put at the side or at the back of the guard-tube, it diminished the sensibility for high potentials of the needle, in fact, exaggerated the very peculiarity in the behaviour of the instrument which is under investigation. This makes it almost certain that the reason why, under ordinary circumstances, a high potential of the needle does not produce a high sensibility, lies in the fact that the guard-tube is not continuous all round in the neighbourhood of the needle.

The bending up which may be seen at the ends of the curves on Sheet V. (Plate 11), as well as at the ends of many of the curves on the preceding sheets, is probably due to a slight tilting of the needle at high potentials. Such a tilting brings the ends of the needle near the quadrants, and much increases the sensibility. (See Section IX., "Sketch of the Mathematical Theory of the White Electrometer.")

TABLE VI.—December 6, 1887. Thin Piece of Aluminium Foil at the side of the Guard-Tube.

Deflecting P.D. in volts.	Potential of needle in volts.	Electrical zero.	Double deflection per Leclanché cell.	Sensibility factor.	Remarks.
1·45	359	— 1	72	47·8	Adjustable quadrant at — 50.
	717	— 5	130	43·2	
	1076	— 7	174	38·6	
	1434	— 13	186	30·8	
	1615	— 17	189	27·9	
	1793	— 23	188	25·0	
	1972	— 29	186	22·5	Points marked × on the curve, Sheet V. (Plate 11).
	2152	— 38	183	20·3	
	2331	— 43	181	18·5	
	2510	— 51	177	16·8	
	2689	— 60	173	15·4	
	2869	— 77	173	14·4	
	3043	— 93	170	13·3	
	3227	— 122	176	12·9	
	3407	— 155	184	12·9	
	3586	— 218	223*	14·8	* This observation confirmed many times.
	717	— 5	140	46·5	
	1434	— 23	186	30·8	
	2152	— 53	183	20·3	
	2869	— 98	173	14·3	
	3048	— 119	172	13·4	
	3227	— 124	172	12·7	
	3406	— 157	173	12·1	
	3586	— 228	221	14·7	

TABLE VII.—December 7, 1887. Thin Piece of Aluminium Foil at the back of the Guard-Tube.

Deflecting P.D. in volts.	Potential of needle in volts.	Electrical zero.	Double Deflection per Leclanché cell.	Sensibility factor.	Remarks.
1·45	359	4	82	54·5	Adjustable quadrant at + 50.
	717	11	130	43·2	
	1076	21	160	35·5	
	1255	26	167	31·7	
	1434	29	172	28·6	
	1615	30	171	25·2	
	1793	31	170	22·6	
	1972	29	166	20·0	
	2152	27	162	17·9	
	2331	23	158	16·1	
	2510	17	153	14·5	
	2689	8	150	13·5	Points marked ◬ on the curve, Sheet V. (Plate 11).
	2869	− 1	145	12·0	
	3048	− 13	141	11·0	
	3227	− 31	137	10·5	
	3407	− 49	138	9·6	
	3586	− 95	142	9·4	
	3765	− 127	152	9·6	
	2987	− 14	142	11·3	
	1972	27	165	19·9	
	1651	33	170	24·5	
	1004	29	157	37·3	
	538	24	114	50·5	
	179	21	53	70·5	Reading on the absolute electrometer not very trustworthy at such low volts.
December 8, 1887.					
					Adjustable quadrant at
1·45	359	− 41	79	52·5	− 50
	717	− 92	134	44·5	− 50
	717	− 51	136	45·2	0
	896	− 61	152	40·4	− 50
	896	− 148	151	40·1	0
	1076	. .	154	34·2	− 50
	1076	. .	162	35·9	0
	1434	. .	170	28·2	− 50
	1434	− 96	168	27·9	0
	717	. .	134	44·5	− 50
	1793	− 240	170	22·6	− 50
	1793	. .	166	22·1	0
	2152	. .	167	18·5	− 50
	2152	− 137	158	17·5	0
	2510	− 165	151	14·3	0
	2869	− 194	145	12·0	0
					Points marked ⊡ on the curve, Sheet V. (Plate 11).

TABLE VIII.—December 8, 1887. Aluminium Foil Removed from the Guard-Tube.

Deflecting P.D. in volts.	Potential of needle in volts.	Electrical zero.	Double deflection per Leclanché cell.	Sensibility factor.	Remarks.
1·45	359	..	82	54·5	Adjustable quadrant at 0
	717	– 30	138	45·9	0
	1076	– 63	175	39·0	0
	1434	..	190	31·6	0
	1793	– 128	194	25·7	0
	2152	– 158	189	20·9	0
	2152	..	203	22·5	+ 50
	2510	– 52	197	18·6	+ 50
	2510	..	187	17·7	0
	2869	..	190	15·7	0
	2869	– 228	187	15·5	+ 50
	1793	..	207	27·5	+ 50
					Points marked ⊙ on the curve, Sheet V. (Plate 11). The points represented by the ▽ sign on the curve, Sheet V., were taken on Dec. 3 and 4. See Table II. and Sheet II.

The two last sets of experiments made with the aluminium foil at the back of the guard-tube, and after the foil had been removed, show that no considerable change is made in the double deflection by turning the micrometer screw attached to the adjustable quadrant through 50 divisions, that is, turning it through 180°.

Some experiments were therefore made to obtain further information on this point, especially to see how a motion of the adjustable quadrant affected the sensibility for different potentials of the needle, and the results are given in Table IX. It will be observed that, for a given potential of the needle, the effect produced on the sensibility, by attaching or altering the position of the foil, is much greater than the effect produced by the corresponding motion of the adjustable quadrant, so that we are justified in regarding the greater part of the difference between the curves on Sheet V. (Plate 11), as really due to the presence and position of the foil on the guard-tube.

Table IX.—December 9, 1887. Effect of Varying the Position of the Adjustable Quadrant. Double Deflection for a Deflecting P.D. of 1·45 Volts.

Potential of needle in volts.	Position of micrometer screw attached to movable quadrant.							
	− 50	− 25	0	+ 25	+ 50	+ 75	+ 100	+ 150
359	84	..	83	..	82	..	83	83
717	..	..	143	..	..	..	149	153
1076	..	..	175	..	182	..	194	
1434	190	..	189	..	200	..	260	
1793	204	193	191	194	207			
2152	..	198	190	189	206			
2510	..	203	185	185	197			
2869	..	..	187	..	189			

V.—Motion of the Electrical Zero.

It was found to be practically impossible to adjust the needle, quadrants, and other parts of the electrometer, so that the electrical and mechanical zeros should be the same for all P.D.'s between the needle and the outside of the instrument, although, when the controlling force was made somewhat large by separating the suspending fibres at the top, it was possible with a careful adjustment to render the difference between the electrical and mechanical nearly nought for all potentials of the needle up to some 1400 volts. For example, on January 30, 1888, when the fibres were separated as far as is possible with the ordinary construction of the White form of quadrant electrometer, and the quadrants and needle very carefully adjusted for symmetry, the electrical zero only differed by three scale divisions from the mechanical, when the needle was gradually charged up from 0 to 1434 volts. On another occasion this motion was reduced to two divisions for the same limits of the potential of the needle, and in a third case there was no perceptible motion of the spot of light on charging up the needle to this potential. But even in these cases there was a considerable motion of the electrical zero for still higher P.D.'s between the needle and the outside case of the electrometer.

On one occasion, when the fibres were very near together, and the controlling force therefore weak, the electrical zero only moved through ten divisions from the mechanical zero when the needle was charged up to as high as 2869 volts. This was, however, a totally exceptional result, and was obtained, as if by accident, after very many trial adjustments. The adjustments of the quadrants, however good they may seem to the eye, are not accurate enough to cause the electrical and mechanical zero to remain constant for a weak control of the needle as the needle is steadily charged up. It does not seem to be sufficient to have only one quadrant adjustable by means of a micrometer screw, and we would suggest that, if two quadrants were provided with a micrometer screw motion, much time would be saved. It is, of course, possible, by moving the adjustable quadrant, to make the electrical zero coincide with the mechanical zero for some one potential of the needle, but although the coincidence is made to occur when the potential of the needle is, say, 2000 volts, it by no means follows that the two zeros will agree when the needle's potential is 1000 volts. And the experiments, of which some of the results are given in the following Table X., and plotted on Sheet VI. (Plate 12), show that when the four quadrants are adjusted symmetrically, as far as the eye can judge, the following result is obtained. First, the electrical zero moves away from the mechanical zero as the potential of the needle is increased; then, having reached a certain distance from the mechanical zero the electrical zero returns towards the mechanical zero as the needle's potential is still further increased; next it passes through the position of the mechanical zero, and finally it moves away farther and farther down the other side of the scale.

TABLE X.—November 8, 1887. Motion of the Electrical Zero. Position of the Electrical Zero Measured from the Position of the Mechanical Zero.

Potential of needle in volts.	Reading on micrometer head attached to movable quadrant.								
	− 100	− 50	0	+ 25	+ 50	+ 75	+ 100	+ 150	+ 200
359	− 52	− 28	− 17	..	− 5	..	9	21	35
538	− 85	− 62	− 31	..	− 4	..	24	54	89
717	− 140	− 100	− 56	..	− 10	+ 15	40	92	150
896	− 200	− 140	− 78	..	− 10	..	63	145	235
1076	− 260	− 183	− 100	..	− 12	..	83	195	
1255	..	− 220	− 122	..	− 10	..	108	250	
1434	..	− 258	− 143	..	− 15	..	120	316	
1793	..	..	− 189	− 100	− 22	+ 63	154		
2152	..	..	− 228	− 135	− 37	+ 60	168		
2510	..	..	..	− 177	− 66	+ 38	158		
2869	..	..	..	− 238	− 110	+ 3	129		

Fibres near at the top. Quadrants about $2\frac{1}{2}$ millims. apart.

VI.—ELECTROMETER USED TO MEASURE POWER.

When the power given by any current to any circuit *ab* (fig. 6) has to be measured by means of a quadrant electrometer the method is as follows :—In series with *ab* is connected a non-inductive circuit *bc*, of resistance *r*; the opposite pairs of quadrants are joined respectively to *a* and *b*, and two observations are taken, one when the needle is joined to *b*, and one when it is joined to *c*. Then the difference between these readings divided by *r* is proportional to the power given to *ab*. A second method consists in connecting the opposite pairs of quadrants respectively with *b* and *c*, and to take two observations, one when the needle is joined to b, and one when it is joined to a. Then, as before, the difference between these two readings divided by r is proportional to the power given to ab.

Fig. 6.

If ab be the primary coil of a transformer, the P.D. between the terminals of which will be 1000 or 2000, or more, volts, the first of the two methods just described of joining up the electrometer will be inapplicable, for, with the ordinary adjustment of this instrument, the spot of light would fly off the scale if one pair of quadrants were connected with a and the other pair and the needle with b, and the P.D. between a and b were some thousands of volts. Hence the second method of joining up the apparatus is alone applicable in such a case.

When the quadrants are joined to b and c, the needle to b, and the outside case of the electrometer to c, the instrument is being used idiostatically. The text-book law

$$D \propto (Q_1 - Q_2) \left\{N - \frac{Q_1 + Q_2}{2}\right\},$$

where D is the deflection, Q_1, Q_2, and N the potentials of the opposite pairs of quadrants, and of the needle relatively to the outside case of the instrument, reduces to

$$D \propto N^2,$$

when the needle is connected with one pair of quadrants, and the outside case with the other pair. Now a P.D. of only some 100 volts between the needle and the outside case will deflect the spot of light to the end of the scale, when the electrometer is used idiostatically. And we can see that the first part of each of the curves on Sheets II., III., and IV. is straight, that is, that the text-book law holds for the quadrant electrometer made by Messrs. White when the P.D. between the needle and the case does not exceed a hundred volts or so. Hence we should anticipate that when this instrument is used idiostatically the deflection would be directly proportioned to the square of the P.D. between the quadrants for any fixed position of the quadrants.

And experiment proves this to be the case, for a number of tests made to check the accuracy of the square law showed that the numbers obtained by dividing the deflection by the square of the potential of the needle did not differ from one another by more than one per cent. for deflections in all parts of the scale. The various P.D.s were obtained and measured by connecting up 200 silver chloride cells with the terminals of a Thomson-Varley sliding P.D. divider, the "zero" point being joined to one pair of quadrants and to the outside of the electrometer, while the "index" was joined to the other pair of quadrants and to the needle.

The square law obtained on using the quadrant electrometer idiostatically is never quite true for small P.D.s, since, as pointed out by Dr. HOPKINSON and the authors some years ago, the contact P.D. between the needle and the quadrants causes the deflection to be different for the same P.D. between the quadrants, depending on whether the needle is charged positively or negatively.

When making the second observation with the second method of joining up the apparatus for measuring the power given to *ab*, we have the pairs of quadrants joined respectively to *b* and *c*, between which there may be a P.D. of 1 or 2 volts, and the needle joined to *a*, the potential of which, when *ab* is the primary of a transformer, may differ by some thousands of volts from that of the quadrants or of the outside of the Leyden jar. In other words, we are now using the electrometer heterostatically in exactly the way in which it is ordinarily employed, excepting that, if an alternating or intermittent current be passing through the circuit *abc*, all the potentials will be varying.

Now we have shown that when the P.D. between the needle and the outside case is some hundreds of volts or more, the formula

$$D \propto (Q_1 - Q_2)\left\{N - \frac{Q_1 + Q_2}{2}\right\}$$

does not hold unless the quadrants be in one definite position. Hence the deflection will not measure the mean value of the expression on the right hand side of the last equation unless the quadrants be in this definite position. But the electrometer method of measuring power is based on the assumption that the deflection measures the mean value of

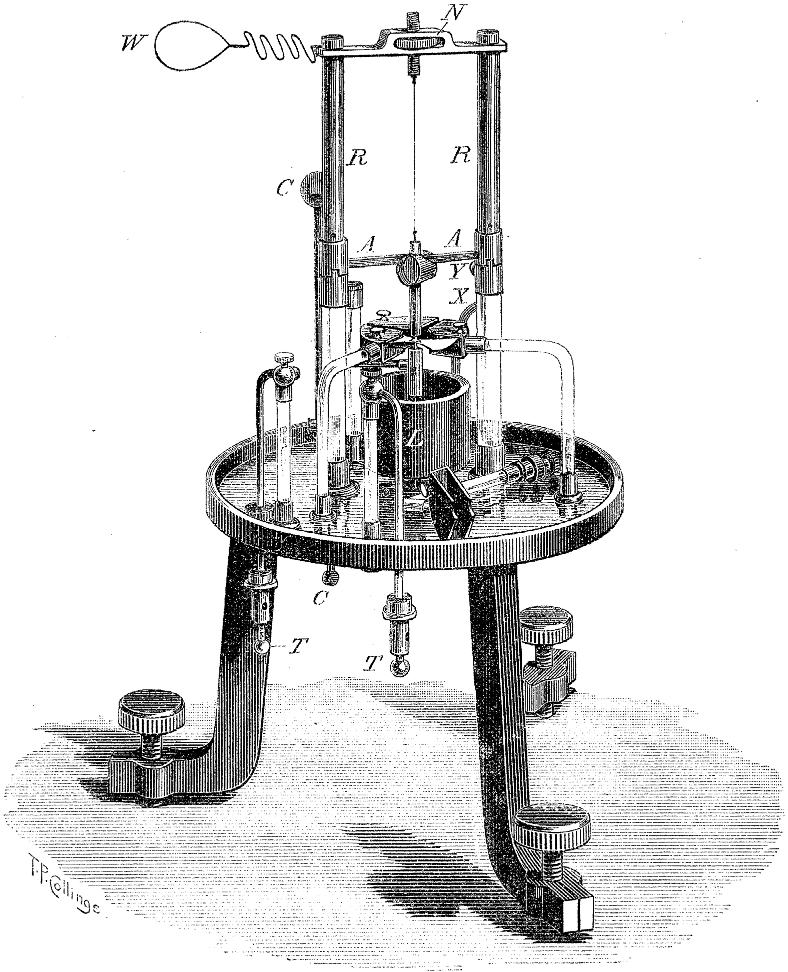
$$(Q_1 - Q_2)\left\{N - \frac{Q_1 + Q_2}{2}\right\}.$$

Hence it follows that *the quadrant electrometer, if measuring the power given to the primary coil of an ordinary transformer, will not give correct results unless the quadrants be in the one special position that we have pointed out.*

It might be thought that, although the electrometer did not obey the usual law given for it, it would be possible to calibrate it empirically as a watt meter. As regards one of the observations, viz., that made when the needle and one pair of quadrants are joined to *b* (fig. 6) and the outside case and the other pair of quadrants to *c*, it is possible to calibrate the electrometer empirically, whatever its law may be. For any special deflection can, for a given adjustment of the instrument, be only produced by one special P.D. between the quadrants. But on making the second observation when the quadrants are joined respectively to *b* and *c* and the needle to *a* the same deflection may be obtained with many different potentials of the points *a*, *b*, and *c*. Hence it follows, that unless the deflection is connected with the P.D.

between the quadrants and the P.D. between the needle and case by a law of a certain special form, it will be impossible to calibrate the quadrant electrometer empirically as a watt meter.

Fig. 7.



Improved Quadrant Electrometer. Glass shade (which is also the Leyden jar) and the Magnetic Control removed.

VII.—Improved Quadrant Electrometer, Construction.

The first condition to be fulfilled in designing an electrometer, indeed, we venture to think in designing any electrical measuring instrument, consists in supporting *all* the working parts from the base, so that on removing the cover, all the parts can be got at and adjusted *in position*. We, therefore, commenced by turning Sir William Thomson's electrometer upside down, so to say, supporting the quadrants, needle, charging rod, &c., from the base, and using the glass shade which covers up our instrument as the Leyden jar.

Fig. 7 shows an instrument with this glass shade removed, the springy wire W being for the purpose of making connection with the interior of the Leyden jar, and it will be seen that every part of the interior can be easily got at without interfering with the adjustment and without having to first clamp the needle, a precaution necessary to be adopted in the quadrant electrometer as made by Messrs. WHITE. Further, not merely are the insulating stems that support the needle and the quadrants protected from dust and moisture, when the cover is on, but also the insulating stems that support the brass terminals, *T*, *T* and the charging rod *CC*. Hence there are no exposed ebonite stems requiring daily rubbing to remove dust and moisture, an operation that not unfrequently electrifies them, resulting in a slow charging of the quadrants when they are insulated. When the instrument is not in use the holes in the base plate through which pass the terminal rods *T*, *T* are closed by means of brass plugs sliding on the terminal rods and pushed up from below.

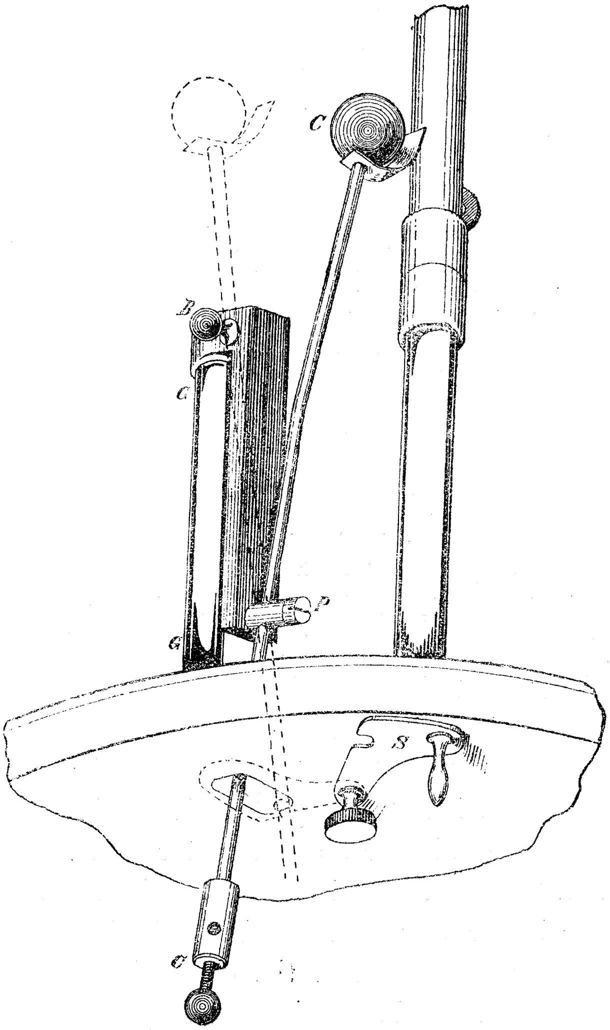


Fig. 8.

In the WHITE form of electrometer the charging rod can be electrically disconnected from the interior by the short ebonite tube, which holds it, being turned by the fingers. To do this, however, the ebonite tube must be tightly grasped in one's fingers, and as they are necessarily chemically dirty, there is the possibility of the Leyden jar being partially discharged in the act of disconnecting the charging rod. This objection is entirely overcome with the arrangement, seen enlarged in Fig. 8, which we have adopted for the charging rod in the improved quadrant electrometer. At the top of a glass rod *GG* is cemented a cap, to which is attached a piece of brass, to the lower end of which is fixed a pivot *P*, round which the charging rod *CC* can turn. The ball *C* at the upper end of this rod is made of lead, and its weight causes the rod to remain in either the position shown by the full line, or by the dotted line. When in the position shown by the full line, the ball *C* is in electric contact with the needle, guard-tube, and the interior of the Leyden jar, the area of contact being increased by the bending of the little piece of flat spring fastened to the ball. The Leyden jar can now be charged, and at the moment that the right amount of charge has been given, the charging rod *CC* is tilted over into the position shown by the dotted line, and the electric connection is broken. This tilting over is easily done by pushing the lower end of the rod with the electrophorus, if an electrophorus be used to charge with, or by means of a glass rod, or by slightly pulling the wire attached to the binding screw at the lower end of *CC*, if such a wire be used to charge through. The slide *S* is now closed to exclude dust and moisture.

When this electrometer is employed for alternating current work it is desirable to connect the binding screw *B* by means of a fine wire with a binding screw attached to the interior of the instrument, so that the electric contact between the charging rod and the interior of the instrument (and which should be very good for very rapid reversals of the charge) does not depend solely on the pressure of the leaden ball at the top of *CC*. To make such a wire connection would be practically impossible with the WHITE form of the quadrant electrometer, since the charging rod must be bodily removed from the instrument before the part of it inside the electrometer can be got at. It is also desirable, when an electrometer is employed for measuring a rapidly alternating P.D., that the electric connection between the electrodes and the quadrants should be far better than can be obtained with the comparatively slight pressure that exists between the ends of the electrodes and the quadrants in the WHITE form of instrument; hence in the improved quadrant electrometer the connection with the quadrants is made by means of screwed contacts. Further, for alternate current work it is not only important to have as good contacts as possible, but also as little capacity as possible in the electrometer. This result we attain by removing the springy wire *W*, and electrically connecting the interior metallic coating of the glass shade with the exterior.

The next point to consider in designing this instrument was the size to give to the needle and quadrants. Theoretical reasoning, confirmed by some roughly made experiments, showed that the sensibility of the quadrant electrometer, as made by Messrs. WHITE, could be much increased if the needle and the quadrants were made smaller than in that instrument. We therefore made the diameter of the quadrants, their vertical depth, and the length of the needle about six-tenths of the size adopted in the White pattern. Feeling, however, that the needle in the White pattern was not broad enough at the widest part to prevent its edges approaching too near the intersection of the quadrants when the needle was deflected, we thought it better not to reduce the width of our needle in the same proportion as its length. Therefore while we reduced its length to six-tenths we left its breadth eight-tenths of the sizes adopted in the White electrometer.

Then as regards the controlling force. In our laboratories are electrometers with a bifilar suspension control, a torsional control, and a magnetic control respectively, and of these three forms we have found that the magnetic control is the best to be used in an electrometer, whose sensibility we require to be able to easily vary. The bifilar suspension control is unnecessarily troublesome, arising partly from the frequent necessity for adjusting for equal tension in the two fibres. For, in the White form of quadrant electrometer, no alteration in the centering of the needle, or in the height of the needle, or in the distance between the fibres, can be effected without a fresh adjustment being rendered necessary for equality of tension in the fibres.

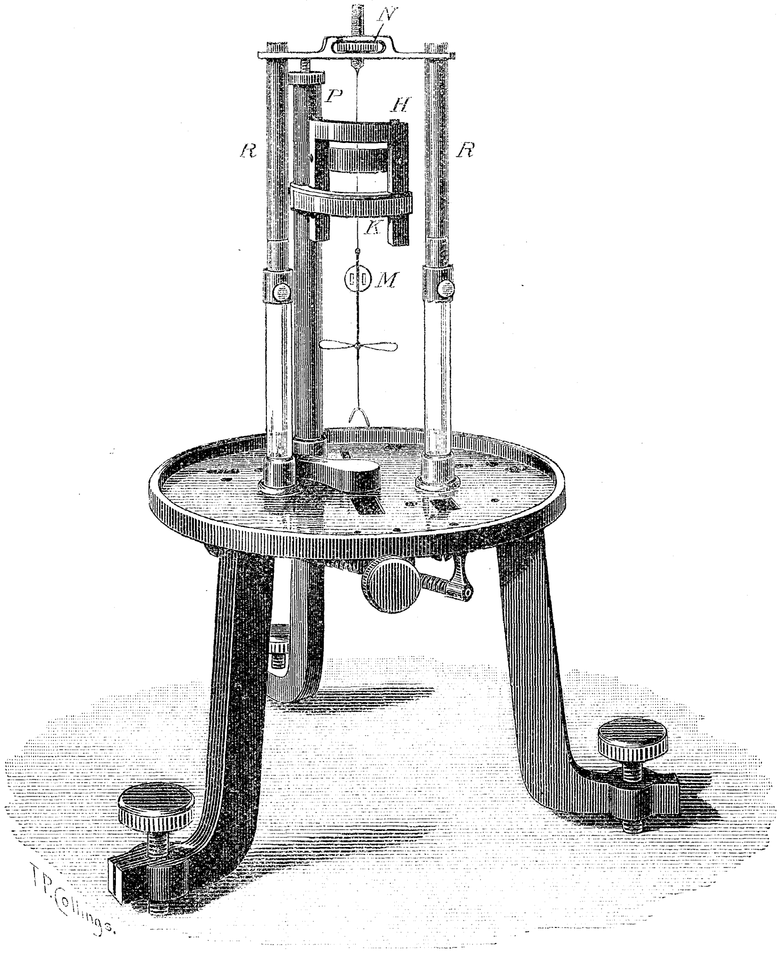
The magnetic control has no doubt the defect that the position of the needle is liable to be affected by outside magnetic disturbance. But this source of error can be practically annihilated if the magnet attached to the suspension wire *be very short and be very weakly magnetised*, and if the requisite magnetic couple be produced by the use of a *very powerful controlling magnet* placed near to the suspended system, and we venture to think that it was a neglect of this principle that caused the abandonment of the magnetic control originally employed by Sir WILLIAM THOMSON in his quadrant electrometers. Another plan that we have found very successful is to fix on the suspension wire an *astatic* combination of magnets, and to place the controlling magnet nearer one of the little magnets than the other.

But probably the most perfect plan is to use, as the astatic combination, two little *vertical* magnets, which may be conveniently attached to the back of the mirror, M, as seen in fig. 9, since *such a combination remains necessarily astatic as far as rotation round a vertical axis is concerned, no matter how much one or both of the magnets may weaken with time.*

Where completeness in the electrometer is of more importance than the expense, the controlling magnet may be a vertical horseshoe carried inside the instrument from a support attached to the base, as seen in fig. 9, which shows the system of magnetic control adopted in the improved electrometer. Two adjustments are provided, one worked by a milled head at the base of the supporting pillar, P, which projects about an inch through the base plate to enable this milled head to be easily turned; the other by a tangent screw fixed under the base plate. Turning the milled head at the base of the supporting pillar slides the soft-iron keeper, K, over the poles of the horseshoe magnet, H, and enables the strength of the controlling field to be varied within wide limits; while, on the other hand, a rotation of the tangent screw causes the supporting pillar with the horseshoe magnet and the keeper which it carries to rotate about the central axis of the instrument, and so enables any variation of the zero to be effected.

Lastly, we supported the lower part of the guard-tube quite independently of the upper part, and hence the objectionable connecting pieces I, I' (fig. 3) are entirely dispensed with, and the result is that, whether the quadrants be pushed far in, or pulled far out, this instrument obeys the recognised formula for a quadrant electrometer.

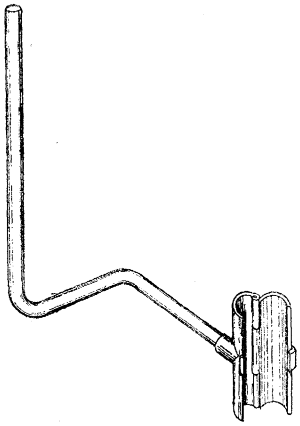
Fig. 9.



Improved Quadrant Electrometer showing the Magnetic Control and the arrangements for varying the strength of the field, also the direction of the field. Quadrants &c., removed.

The brass rods, R, R (fig. 7), which are each supported on a glass rod, carry all that portion of the apparatus that is at the potential of the needle, except, of course, the interior of the Leyden jar glass-shade. The needle hangs by means of a silk fibre from a screw which can be raised or lowered by turning the nut *N*, a vertical slot in the screw and a horizontal pin projecting from the framework preventing the screw itself turning round. The platinum wire at the bottom of the needle dips into some sulphuric acid contained in a leaden cup *L*. This cup is supported and electrically connected to *RR* by a stout wire *X*, the upper end of which slides through a clamping screw *Y* at the back of the lower end of the right-hand brass rod *R*. The leaden cup can, therefore, be easily raised or lowered and also centred. Similarly the lower portion of the guard tube is carried by a stout wire passing through a clamping screw at the back of the lower end of the left-hand brass rod *R*. This portion of the guard-tube is hinged and is shown open in fig. 10. After being put in position the tube is closed round the platinum wire, which projects down from the needle but so as not, of course, to touch this wire.

Fig. 10.



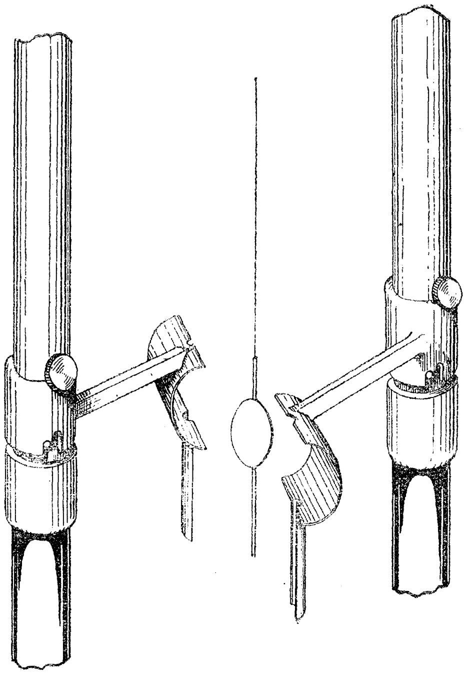
The upper portion of the guard-tube is made in two halves which are carried by the arms *A*, *A* (fig. 7) attached to collars sliding with a bayonet-joint motion on the rods *R*, *R*. By raising up these collars and turning them round the two halves of the upper portion of the guard-tube can be opened as seen in fig. 11, and the mirror and the wire attached to it got at. In fact, by opening and turning aside both the lower and the upper portions of the guard-tube, the mirror, needle, and its attached wire, can be left hanging freely from the silk fibre suspension, and it can be seen whether they are hanging properly, or whether they require adjustment.

The two upper portions of the guard-tube are made skew, as shown in fig. 11, in order that when the mirror is parallel to the needle, and the latter symmetrical with the quadrants, the glass supports of the latter may clear the glass supports of *R*, *R*.

To avoid confusion in the illustrations the fine wire spirals connecting the opposite pairs of quadrants together, and with the terminal screws at the tops of the glass rods supporting the terminal rods *T*, *T*, have been omitted.

We have to thank Mr. Mather for taking a leading part in designing and constructing this improved quadrant electrometer.

Fig. 11.



VIII.—RESULTS OBTAINED WITH THE IMPROVED QUADRANT ELECTROMETER.

A large number of experiments have been made with the improved quadrant electrometer, and marked differences are observed between its behaviour and the behaviour of the White instrument.

First, altering the distance between the quadrants of our instrument does not materially affect the sensibility. For example, increasing the distance between adjacent quadrants from $2\frac{1}{2}$ to 4 millims. hardly varies the sensibility at all; whereas with the White electrometer, drawing out the quadrants increases the sensibility (a result which, as we have pointed out, is quite opposed to the text-book formula for a quadrant electrometer), and this increase in sensibility is brought about by a complete change in the shape of the curve connecting the potential of the needle with the deflection produced by a given P.D. between the quadrants.

Secondly, whereas with the White instrument it is only with one position of the quadrants that the potential of the needle is even approximately proportioned to the deflection produced by a given P.D. between the quadrants, this result is obtained with considerable accuracy with *any* symmetrical position of the quadrants in our instrument.

The following table is a sample of the results obtained, no special adjustment of the quadrants having been made to obtain constancy in the sensibility factor. This sensibility factor, as already explained, is the ratio of the deflection, in microradians, of the reflected ray of light to the product obtained by multiplying together the deflecting P.D. in volts, and the P.D. between the needle and the outside case also in volts.

TABLE XI.—July 13, 1888. Improved Electrometer.

Deflecting P.D. in volts.	Potential of needle in volts.	Electrical zero.	Double deflection per Leclanché cell.	Sensibility factor.	Remarks.
4·35	359	1	13·5	9·0	Strong magnetic controlling field
	717	1·5	26·2	8·7	
	1,076	4	39·0	8·6	
	1,434	14	54·2	9·0	
	1,793	34	69·0	9·2	
	2,152	92	87·8	9·7	
	1,434	36	53·7	8·9	
	717	8	26·0	8·6	
	359	4	13·8	9·2	
	179	3	7·0	9·3	

Further, it is possible to practically obtain far greater sensibility with this improved electrometer than with the specimen we possess of the quadrant electrometer, as made by Messrs. WHITE. The greatest double deflection for a Leclanché cell that we have been able to obtain, with our specimen of the White pattern, is 386 scale divisions, the P.D. between the needle and the outside case being 2500 volts, the scale being at 5 feet distance from the mirror, and 1 foot length of scale containing 290 divisions. It would, however, be impossible to work with this adjustment, as the instability was too great. Practically, therefore, about 250 scale divisions, as a double deflection for a Leclanché cell, or, what is the same thing, about 180 divisions per volt P.D. between the quadrants, is all that we can obtain with our specimen of the White electrometer.

With the improved form, on the contrary, very great sensibility indeed can be obtained by weakening the controlling magnetic field, the only limit being caused by the increased importance of want of symmetry, the greater disturbing effects produced by capillary attraction, by the motion of exterior magnetic bodies, &c., as the sensibility is made greater and greater. When the controlling field was made very weak it was not found advisable to make the P.D. between the needle and the outside case more than 1800 volts. Under these circumstances *it was possible to obtain, as a double deflection,* 100 *scale divisions for a P.D. between the quadrants of only one hundredth part of the E.M.F. of a Leclanché cell* corresponding with a sensibility factor of 580. The needle, it is true, was very unstable, and the deflection was not definite enough for ordinary practical use. Possibly, however, the instrument might be used very well in this condition for null methods when great sensibility is required.

However, with a somewhat less sensibility, a double deflection of 44 divisions being obtained for one hundredth of the E.M.F. of a Leclanché cell, the improved electrometer worked more satisfactorily than did the White instrument, when adjusted to have its maximum sensibility, so that there is no doubt that the improved electrometer is at least ten times as sensitive as our specimen of the White pattern, when the two instruments are adjusted to be in an equally trustworthy condition as regards definiteness of the zero, definiteness of a given deflection, &c.

Tests were also made of the two instruments when used idiostatically; and, as regards the White pattern, it was found, as stated by Professor GRAY on p. 299, vol. 1, 'Absolute Measurements in Electricity and Magnetism,' that "The quadrant electrometer may be used idiostatically for the measurement of differences of potential of not less than 30 volts." But, on the contrary, we found that three volts would give a double deflection of about 50 scale divisions with the improved electrometer when used idiostatically. *Hence the improved instrument is also ten times as sensitive as the White pattern when used idiostatically.*

Spring, 1891.—In what precedes we have shown that the improved electrometer is free from the defect in the White pattern, which forms the subject of this paper. We have attributed this defect to the electrical action of certain pieces *I*, *I′* (fig. 3), which connect the upper and lower portions of the guard-tube in the White pattern. But as our instrument differs in many points from the White pattern, not possessing, for example, a bifilar suspension which, according to Dr. HOPKINSON, is a necessary feature in the electrometer in order to explain the major part of the anomalous action of the White instrument, we determined not to publish this paper until we had experimentally proved, beyond all doubt, that our explanation was correct. This we were unable to do until the year 1891, various reasons having prevented our taking up the electrometer investigation again from the point we left it some years ago.

First, as regards a bifilar suspension being necessary, in order that the deflection produced by a given P.D. between the quadrants should be very far from proportional to the P.D. between the needle and the outside case. This question is settled by the experiments that we have made this year on a quadrant electrometer made by Messrs. WHITE, in the possession of the Royal College of Science, and the needle of which has been suspended with a single quartz fibre by Professor BOYS. The following were the results that we obtained, the quadrants being about 2 millims. apart, and the instrument in excellent condition in so far that the spot of light returned absolutely to zero, and a deflection could be read with confidence to $\frac{1}{10}$th millim. The slight irregularities in the curve on Sheet VII. (Plate 11), which graphically represents the results contained in the following Table XII., arise from the potential of the needle having been read by means of a portable pivotted electrostatic voltmeter, the suspension of which was, of course, far more frictional than the quartz fibre of the quadrant electrometer. This voltmeter was previously calibrated by comparison with the absolute electrometer.

TABLE XII.—February 3rd, 1891. White Electrometer with Single Quartz Fibre Suspension.

Deflective P.D. in volts.	Potential of needle in volts.	Electrical zero.	Deflection per one volt.	Sensibility factor.
0·5	0	23·9	..	..
	260	30·6	134	510
	300	31·5	152	510
	335	32·8	178	530
	370	34·1	204	550
	390	36·2	246	630
	410	37·7	276	670
	427	39·4	310	720
	437	40·3	328	750
	440	40·4	330	750

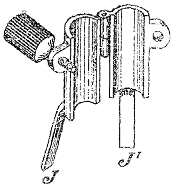
From the preceding we see that even when the needle is suspended by a single fibre, an increase in the potential of the needle from 260 to 440 volts alters the sensibility factor by 50 per cent. It is, therefore, quite certain that the peculiar action in the White electrometer can occur without a bifilar suspension.

Next we had made the collar, or saddle, shown in fig. 12, which could be fastened on the lower part of the upper guard-tube of the improved electrometer, the pieces *J*, *J′*, projecting down into the quadrants on each side of the needle. As these projections *J*, *J′*, are at the same potential as the needle, there will be the same sort of electrical action between them and the needle as there is between the pieces *I*, *I′* (fig. 3) of the White instrument. Hence, if our explanation of the defect in the White instrument be correct, the application of the collar with its projections *J*, *J′* (fig. 12), to the guard-tube of the improved electrometer ought to destroy the straight-line-law which the numbers given in Table XI. show that this instrument possesses.

In Table XIII. are given the results obtained with the improved electrometer just before attaching the saddle piece (fig. 12), while those given in Table XIV. were obtained after merely attaching the saddle (fig. 12), no other change whatever being made in the instrument.

The adjustment of the quadrants, and the position of the controlling magnet were quite different when obtaining the results given in Table XIII. from what they were when the numbers given in Table XI. were obtained nearly three years ago with the same instrument. The sensibility factor had, therefore, quite a different value in March, 1891, from what it had in July, 1888, but, just as a large variation in the potential of the needle caused at that date practically no change in the sensibility factor with the improved electrometer, so now the sensibility factor is practically independent of the potential of the needle before attaching the saddle. After, however, the saddle has been attached, we see first that the sensibility factor varies rapidly with the potential of the needle, and, secondly, that it is much less than before the saddle was attached, its nearly constant value (22), obtained before the saddle was used, being only reached with the saddle on when the potential of the needle has been lowered to some 200 volts.

Fig. 12.



The results given in the Tables XIII. and XIV. are plotted in the curves shown on Sheet VIII. (Plate 12), and prove without doubt that the pieces *I*, *I′* (fig. 3), in the White electrometer are the cause of all the trouble.

TABLE XIII.—March 20, 1891. Improved Electrometer before Attachment of Saddle (fig. 12).

Deflecting P.D. in volts.	Potential of needle in volts.	Electrical zero.	Double deflection per Leclanché cell.	Sensibility factor.
1·45	1250	+ 29	104	21·5
	1170	+ 29	99	21·8
	905	+ 2	75·5	21·5
	860	+ 1	72	21·5
	812	0	68·5	21·8
	740	— 3	61·5	21·5
	700	— 2	59	21·8
	660	— 1	55·5	21·7
	579	— 1	48	21·3
	505	— 2	43	22
	415	— 3	34·5	21·5
	340	— 2	29	22
	1305	— 40	111	21·8
	1090	— 30	93·5	22
	1140	— 42	97	22
	915	— 29	75·5	21·3
	860	— 23	72·5	21·8
	965	— 34	79·5	21·3
	1015	— 28	85·5	21·7
	1275	— 63	108·5	21·8
	1152	— 54	97	21·7
	1277	— 68	110·5	22·2
	1235	— 65	107	22·4
	1140	— 53	99	22·4
	1087	— 49	93·5	22·2

TABLE XIV.—March 20, 1891. Improved Electrometer, Saddle Attached.

Deflecting P.D. in volts.	Potential of needle in volts.	Electrical zero.	Double deflection per Leclanché cell.	Sensibility factor.
1·45	1430	240	28·5	5·1
	1330	232	30	5·9
	1260	222	31·5	6·5
	1160	207	32·5	7·2
	1005	185	35	9·0
	955	173	36	9·7
	910	164	35	9·9
	850	153	35	10·6
	795	143	35·5	11·6
	705	128	34·5	12·7
	670	117	33·5	13·3
	610	106	33	14
	570	99	32·5	14·8
	530	89	31·5	15·4
	490	80	31	16·4
	450	71	29·5	17·1
	400	59	27·5	17·8
	355	50	25·5	19·2
	305	41	23·5	19·9
	260	32	21	20·8
	210	23	18	22·3
	165	17	14·5	22·7

As the electrical zero altered very much in the above set of measurements made with the saddle attached, experiments were subsequently tried to see whether this alteration of zero affected the sensibility. In each of the above cases, the controlling magnet was turned so as to alter the direction of the controlling field without altering its strength, and, as it was found that the change of zero thus produced did not materially alter the sensibility for any one of the potentials of the needle given in the above table, it was inferred that the mere alterations of the zero mentioned in the table did not materially affect the sensibilities.

[Lastly, an experiment was made to try how inaccurate the improved quadrant electrometer might be, if no care were taken to adjust the quadrants even for symmetry, and if the needle were suspended with a very coarse silk fibre, which had neither been boiled nor subsequently cleaned with absolute alcohol, in accordance with the plan that we usually adopt with silk fibres for suspension. The keeper, *K*, of the horseshoe magnet *H* (fig. 9) was placed so that the magnetic control was a maximum, and the instrument was used idiostatically. In order that, although the electrometer was used idiostatically, the deflection might be reversed and zero errors eliminated, the needle and the outside case were attached respectively to two of the terminals of a reversing key, and the pair of quadrants with the other two terminals respectively.

The results obtained are given in Table XV., and, considering that the instrument was set up in the rough way above described, the constancy of the sensibility factor is sufficiently striking.

TABLE XV.—July 1891. Improved Quadrant Electrometer used Idiostatically.

Reading of potential divider.	P.D. between quadrants, in volts.	Double deflection of reflected ray, in scale divisions.	Sensibility factor, or double deflection of needle, in micro-radians divided by square of P.D., in volts.
10	17	9	11·0
15	25	20	10·8
20	33·6	34·5	10·58
25	41·9	53·4	10·43
30	50·3	77	10·43
35	58·8	103·6	10·38
40	67	134	10·00
45	75·5	169	9·98
50	84	208·5	9·98
55	92·3	252	9·95
60	100·6	299	9·95
65	109	350	9·93
70	117·5	405·5	9·91
75	126	465	9·91
80	134	529	9·91
85	142	597	9·91
90	151	669·5	9·91
70	117·5	405·5	9·91
50	84	207·5	9·93
30	50·3	75·5	10·00
20	33·6	34	10·40

N.B.—The fibre suspending the needle was a very coarse one, and the instrument was tested without any previous adjustment of the quadrants even for symmetry.

One hundred and seventy old silver chloride cells were connected with a Thomson and Varley potential divider. The potential of one hundred of these cells was tested by an electrostatic voltmeter, and found to be 99 volts. One scale division corresponded with an angular movement of the needle equal to $\frac{1}{2900}$ radian.—*July* 20, 1891.]

We cannot conclude this account of the numerous defects of the Thomson quadrant electrometer, as hitherto made by Messrs. WHITE, and of the way to overcome them, without expressing the deep debt of gratitude that we, in common with electricians of all countries, owe to the inventor of the electrometer, for having provided us twenty years ago with an instrument that has been of inestimable value, both for scientific research and for the commercial testing of submarine cables.

IX.—SKETCH OF THE MATHEMATICAL THEORY OF THE WHITE ELECTROMETER.

Before we were certain of the cause of the peculiarities in the action of the White electrometer, we devoted a large amount of time to a mathematical investigation of its behaviour. This gave us many weeks' hard work, as the investigation is a complicated one. For, in order to obtain an equation (for the deflection of the needle in terms of its potential and the P.D. of the quadrants) that would fit all the various curves obtained experimentally, we found it necessary to take into account:—

1st. The repulsion between the pieces I, I' (fig. 3), of the guard-tube and the needle;

2nd. The diminution of this repulsion as the quadrants were separated symmetrically;

3rd. The alteration that the tilting of the needle at high potentials produces on this repulsion;

4th. The alteration that the tilting of the needle produces on the rate of variation, with motion of the needle, of the coefficient of induction between the insulated pair of quadrants and the needle.

As, however, subsequently to this mathematical work, we completed the experimental proof already described of the cause of the curious action in the White electrometer, we think that it will be unnecessary to give more than a brief sketch of the mathematical investigation.

Let N be the P.D. between the outside of the Leyden jar of the White electrometer and the system of conductors, consisting of the needle, guard-tube, sulphuric acid, &c. Let $2n$ be the capacity of this system of conductors. Let the pair of quadrants that is connected with the outside of the Leyden jar be regarded as being at potential nought, and let V be the potential of the other pair. Let $2q$ be the capacity of the insulated pair of quadrants, and s the coefficient of induction between this pair and the needle. Then the energy stored up in the system is

$$\mathrm{N}^2 n + \mathrm{V}^2 q + \mathrm{NV}s,$$

therefore, if $m\theta$ be the moment exerted by the bifilar suspension, when the needle is deflected through an angle θ from the equilibrium position,

$$m\theta = \mathrm{N}^2 \frac{dn}{d\theta} + \mathrm{V}^2 \frac{dq}{d\theta} + \mathrm{NV} \frac{ds}{d\theta}.$$

Now, it is usually assumed that, in consequence of symmetry,

$$\frac{ds}{d\theta} = 2\frac{dq}{d\theta} = k, \text{ a constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

and

$$\frac{dn}{d\theta} = 0,$$

which leads to the ordinary equation,

$$m\theta = k\mathrm{V}\left(\mathrm{N} - \frac{\mathrm{V}}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

The repulsion between the projections I, I' (fig. 3), of the guard-tube would not necessarily render assumption (1) unjustifiable, since the rate of variation of the capacity of the insulated pair of quadrants, and the rate of variation of the coefficient of induction between this pair and the needle, would not necessarily be affected by the action of the guard-tube. But in all probability this action between the projections I, I', and the needle would affect the distribution of electricity on I, I', and on the needle near them as the needle moved, so that the capacity of the system of conductors at potential N would vary somewhat as the needle moved. In other words, $dn/d\theta$ would not equal nought, and the equation for the White electrometer, taking into account this action of the guard-tube, would be

$$m\theta = k\mathrm{V}\left(\mathrm{N} - \frac{\mathrm{V}}{2}\right) + \mathrm{N}^2 \frac{dn}{d\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3),$$

instead of the text-book equation (2).

When the fibres are near together so that the control they exercise is small, experiments show that θ does not increase as fast as it ought to do when N is increased, and that for moderately high values of N, θ actually diminishes instead of increasing. From this it follows that $dn/d\theta$ in equation (3) is negative, a result that might be expected, seeing that, as the needle is deflected, so that portions of it approach the projections I, I' of the guard-tube, it is probable that the action between these projections and the needle at the same potential tends to diminish the capacity of the needle and guard-tube.

Further, it is clear that k diminishes as the quadrants are pulled out symmetrically so as to increase the distance between them; consequently, if (2) were the true formula for the White electrometer, the sensibility ought to diminish as the quadrants are drawn out. But, as seen from our curves on Sheets III. and IV. (Plate 10), and as is probably generally known, the sensibility actually increases as the quadrants are withdrawn symmetrically. Hence $dn/d\theta$ for a given value of θ must diminish numerically as the quadrants are drawn out, and this diminution in the negative value of $dn/d\theta$ must more than compensate for the diminution in k. This is consistent with what we know, since $-\mathrm{N}\, dn/d\theta$ represents the couple exerted between the charge on the needle and the charges on the projections I, I', of the guard-tube, and it might be expected that the repulsion between two conductors at the same fixed potential would be increased by enclosing them in a conductor like the quadrants at a low potential, and diminished as this cover is more and more withdrawn.

Although the electrical zero is made to correspond with the mechanical zero by an adjustment of the fourth quadrant, the position is not necessarily a position in which the electrical attraction of the quadrants on the needle is zero, but is a position in which the action of the guard-tube on the needle is balanced by the action of the quadrants when all are at the same potential. Let α be the angle between the zero position of the needle, and the position in which the force exerted by the guard-tube on the needle is nought, then $\theta - \alpha$ is the angle through which the needle has been moved against the action of the guard-tube on the needle. This force will vary in sign with the sign of $\theta - \alpha$, therefore, if $-dn/d\theta$ be expanded in powers of $\theta - \alpha$, it must be of the form

$$a(\theta - \alpha) + b(\theta - \alpha)^3 + c(\theta - \alpha)^5 + \&c.,$$

so that if $\theta - \alpha$ be small we may say that approximately

$$-\frac{dn}{d\theta} = a(\theta - \alpha),$$

that is

$$m\theta = k\mathrm{V}\left(\mathrm{N} - \frac{\mathrm{V}}{2}\right) - a\mathrm{N}^2(\theta - \alpha),$$

or

$$\theta = \frac{k\mathrm{V}\left(\mathrm{N} - \frac{\mathrm{V}}{2}\right) + a\alpha\mathrm{N}^2}{m + a\mathrm{N}^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

If then ϕ represents the double deflection of the needle obtained by reversing V (the P.D. between the quadrants), and ψ represents the position of the electrical zero, that is, the position of the needle when all four quadrants are at the same potential,

$$\phi = \frac{2k\mathrm{VN} + a\alpha\mathrm{N}^2}{m + a\mathrm{N}^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5),$$

$$\psi = \frac{a\alpha\mathrm{N}^2}{m + a\mathrm{N}^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6),$$

and equations of this form would probably give the value of the double deflection and the electrical zero of the White electrometer, if it were possible to hang the needle so symmetrically that there was no tilting even when it was raised to a high potential.

The tilting of the needle will increase the value of k, and this probably accounts for the rise which occurs at the ends of many of the sensibility curves obtained with the White electrometer and shown on Sheets II., III., IV., and V. Indeed, if there were no action due to the guard-tube it is probable that for all symmetrical distances between the quadrants and for all distances between the fibres the sensibility would increase more rapidly than the potential of the needle, the bending in the sensibility curve being the greater the more easily the needle could be tilted. With the improved quadrant electrometer there is but little tilting of the needle, but in the highest part of the curve, obtained with this instrument when the saddle was not on Sheet VIII. (Plate 12), there is an indication of the sensibility curve bending up towards the vertical axis along which deflection is measured. With the White electrometer we can, in spite of the opposing action of the guard-tube projections I, I' (fig. 3), obtain a sensibility curve concave to the axis along which deflection is reckoned, that is, we can make the sensibility increase more rapidly than the potential of the needle in one or other of two ways. If a bifilar suspension be employed, which tends to resist tilting of the needle, then the quadrants must be drawn out so far that, even for high potentials of the needle, the repulsive action of the guard-tube shall be relatively not large. Examples of this are seen in the curves marked "Quadrants about 4 millims. apart" on Sheets III. and IV. (Plate 10). If a unifilar suspension be employed, which enables tilting to take place much more easily, then a sensibility curve concave to the axis along which deflection is reckoned can be obtained, without drawing the quadrants out far, if the control be so weak that a deflection to the end of the scale is obtained for a potential of the needle too small for the control due to the guard-tube to be important. An example of this is seen in the curve on Sheet VII. (Plate 11), obtained with a White electrometer having its needle suspended by a fine quartz fibre.

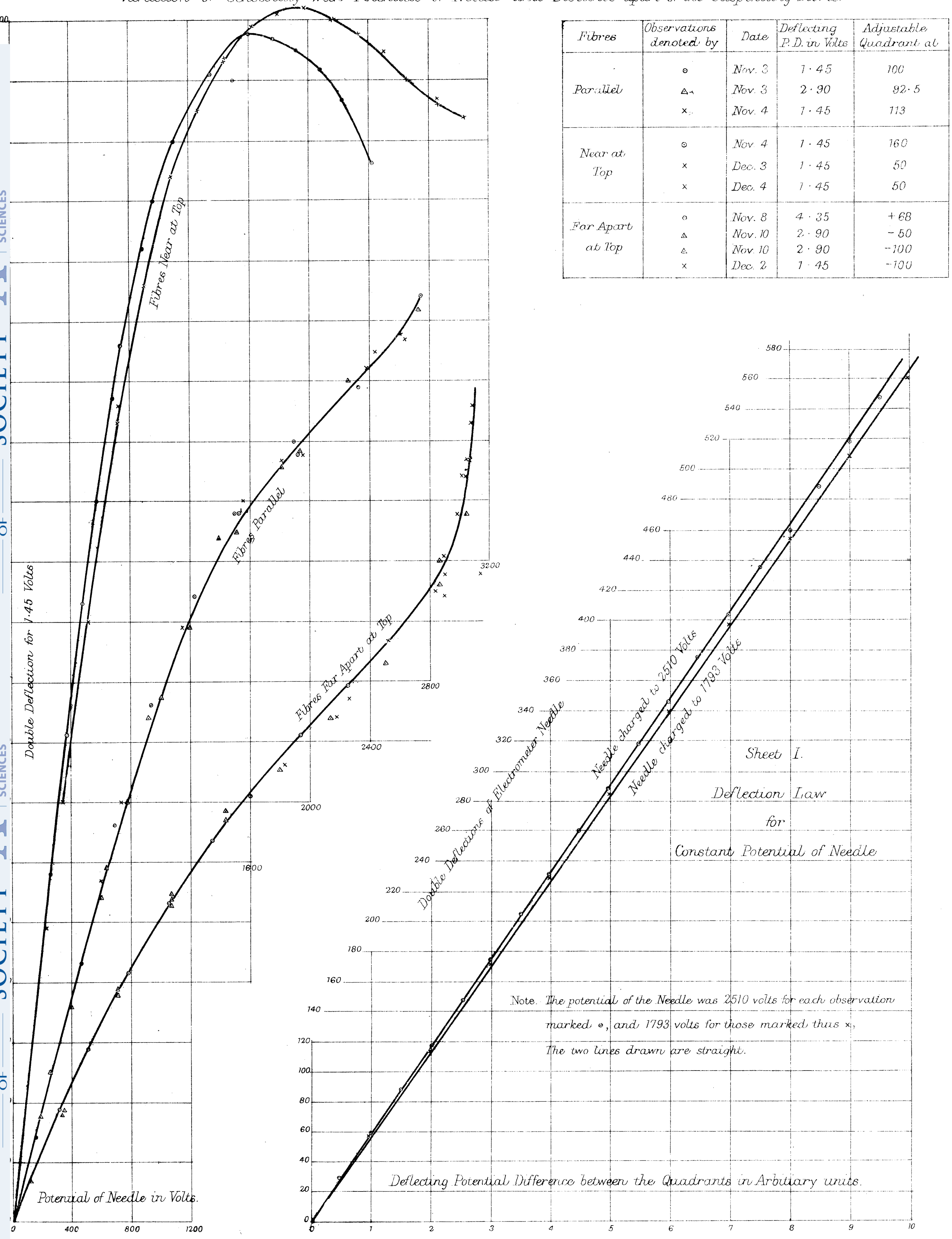
And generally it would appear that for a White electrometer to have a sensibility directly proportional to the potential of the needle, it is necessary to separate the quadrants by such an amount that the increase in the repulsion between the guard-tube and the needle exactly balances the effect due to increased tilting as the potential of the needle is raised, or, in other words, it is necessary that the quadrants should be at such a distance apart that the increase of $N^2 \, dn/d\theta$ (equation 3) exactly balances the effect of increase of k in the expression $kV(N - V/2)$ as N increases.

We had, therefore, to express k, a and $a\alpha$ in equations (5) and (6) as functions of N, and after much working we succeeded in obtaining expressions for ϕ and ψ, each involving the first, second, third, and fourth power of N, and three constants, which agreed very fairly well with the various sensibility curves obtained with the White electrometer and with the curves on Sheet VI. (Plate 12) for the "Motion of the Electrical Zero" as the potential of the needle and the position of the adjustable quadrant were changed.

Sheet II.

Variation of Sensibility with Potential of Needle and Distance apart of the Suspending Fibres.

Fibres	Observations denoted by	Date	Deflecting P. D. in Volts	Adjustable Quadrant at
Parallel	⊙	Nov. 3	1·45	100
	△ +	Nov. 3	2·90	92·5
	×	Nov. 4	1·45	113
Near at Top	⊙	Nov. 4	1·45	160
	×	Dec. 3	1·45	50
	×	Dec. 4	1·45	50
Far Apart at Top	⊙	Nov. 8	4·35	+ 68
	△	Nov. 10	2·90	− 50
	△	Nov. 10	2·90	−100
	×	Dec. 2	1·45	−100



Sheet I.

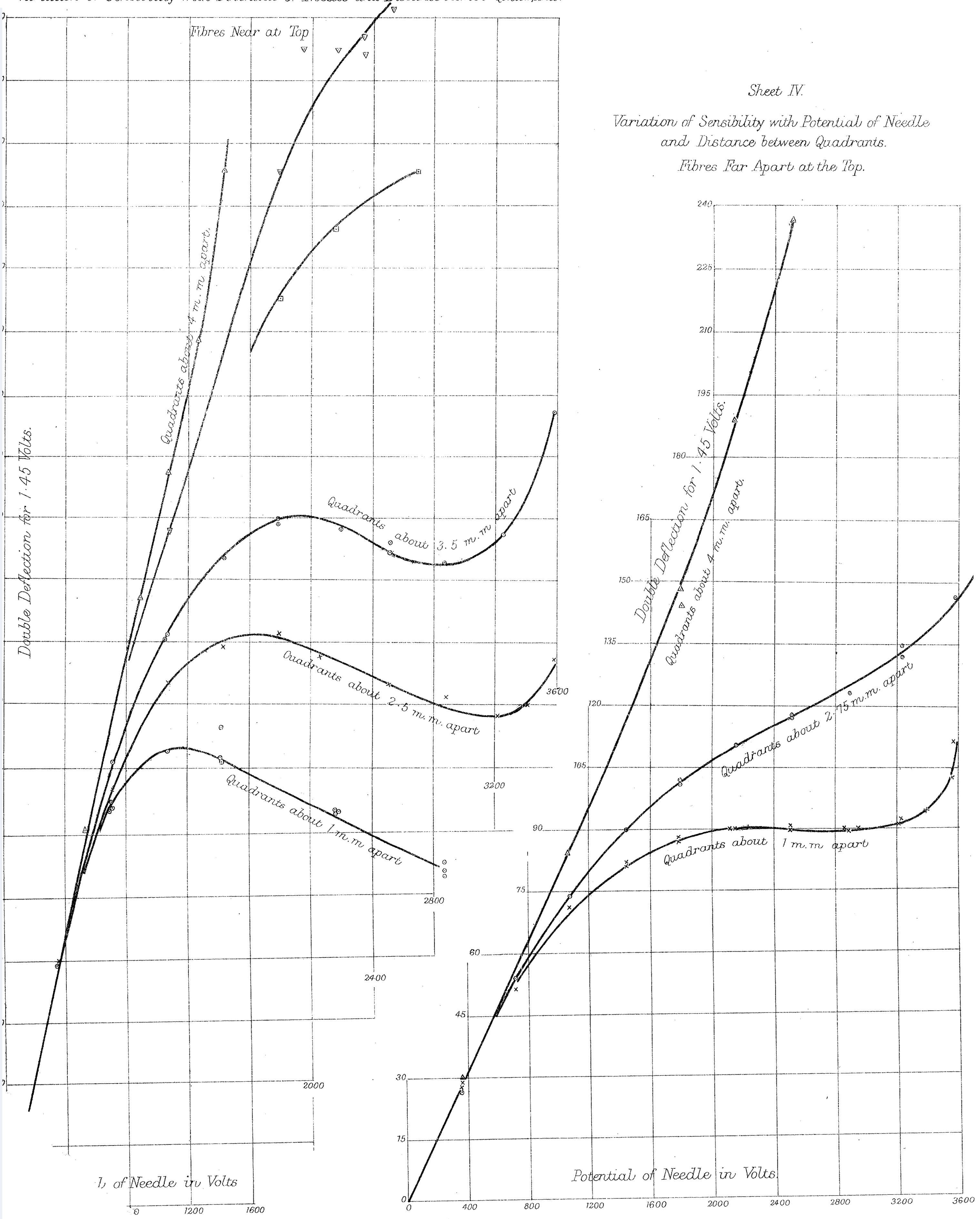
Deflection Law for Constant Potential of Needle

Note. The potential of the Needle was 2510 volts for each observation marked ⊙, and 1793 volts for those marked thus ×.
The two lines drawn are straight.

West, Newman, lith.

Sheet III.

Variation of Sensibility with Potential of Needle and Distance between Quadrants.

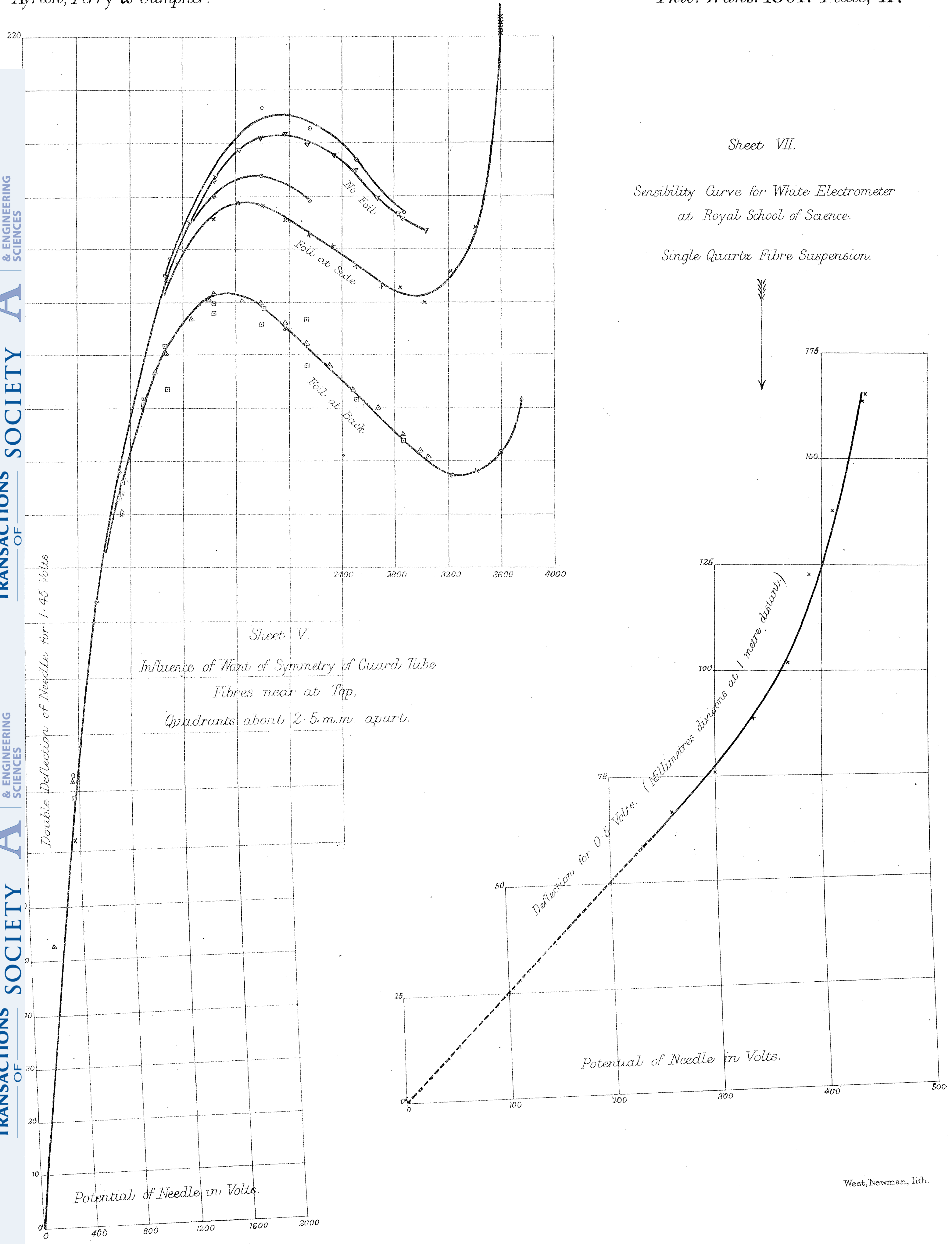


Sheet IV.

Variation of Sensibility with Potential of Needle and Distance between Quadrants.

Fibres Far Apart at the Top.

West, Newman, lith.



Sheet V.

Influence of Want of Symmetry of Guard Tube

Fibres near at Top,

Quadrants about 2·5 m.m. apart.

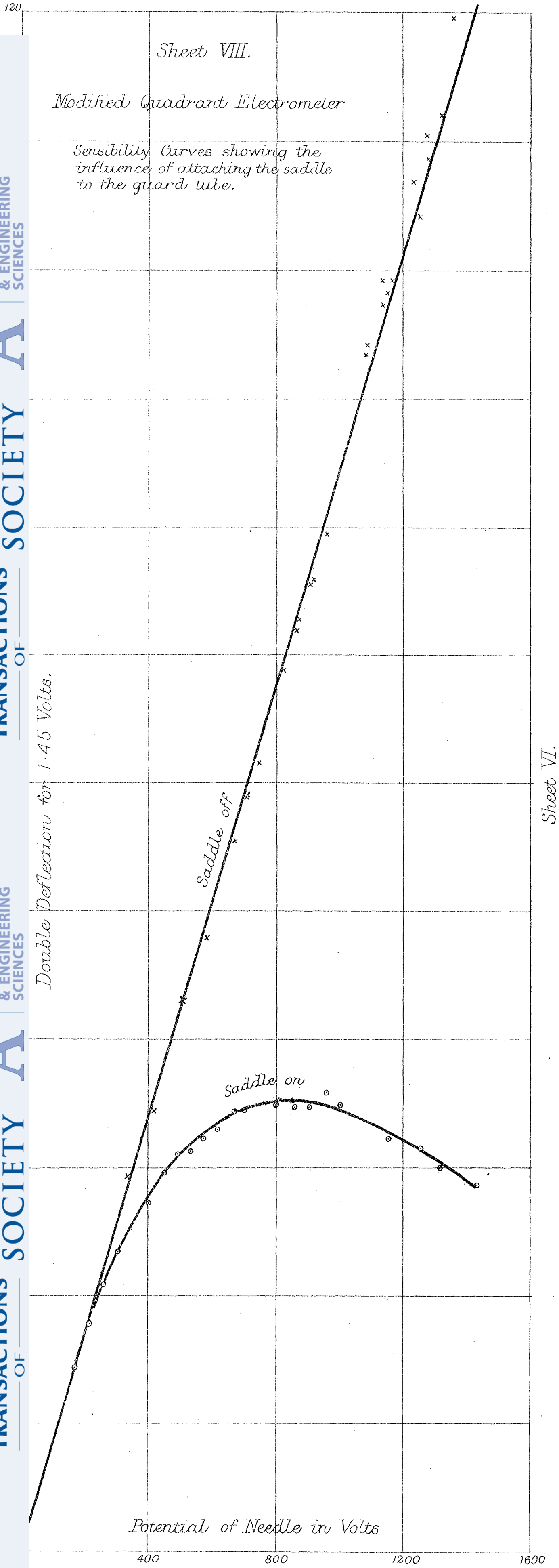
Sheet VII.

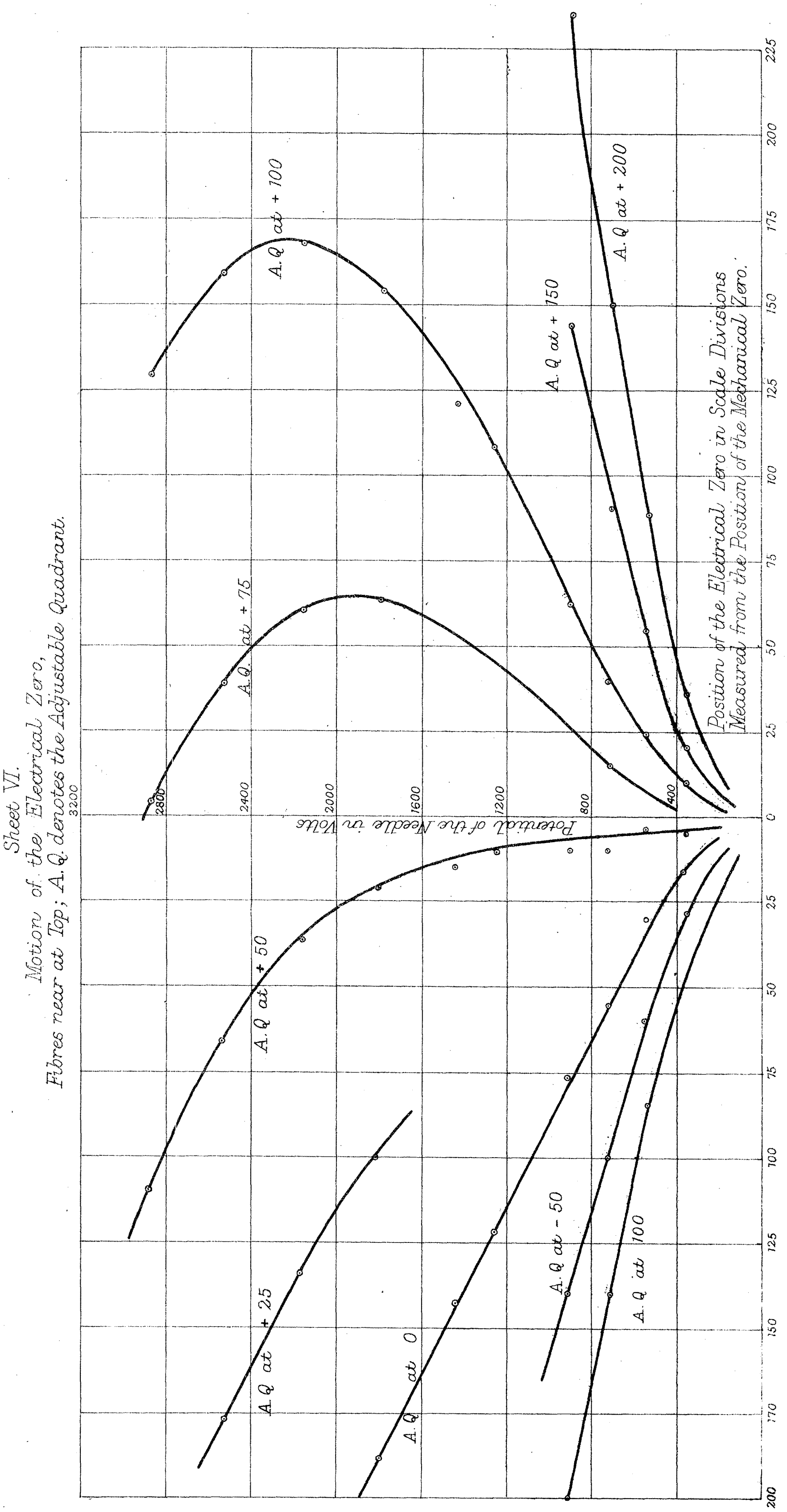
Sensibility Curve for White Electrometer

at Royal School of Science.

Single Quartz Fibre Suspension.

West, Newman, lith.





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